U. S. AIR FORCE

PROJECT RAND

RESEARCH MEMORANDUM

A MODEL OF THE PROCUREMENT-REPAIR

DECISION FOR A SPARE ITEM

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FOREWORD

This study develops a mathematical technique for analyzing the interactions among several factors affecting decisions as to when and how much of an aircraft spare part to buy and repair. This study was undertaken to help develop an understanding of these interrelationships and as a step toward obtaining a systematic technique for making decisions in this important area of requirements determination.

We are concerned with describing the factors affecting decisions as to the amount of each spare part to be procured and repaired, and the amount to be distributed to each base or retained at the depot, and the timing of these actions. Such decisions must be made in a dynamic environment in which aircraft are phasing in or phasing out, spare parts are subject to obsolescence, and the future demand for the part can only be predicted with uncertainty.

This technique is intended for use with high value items only (e.g., items priced over $500, or of much overall dollar importance), because it requires a large amount of computation. RAND is working on other techniques to be used with lower-priced or less significant spare items.[1]

The technique includes many of the important factors involved, but in order to handle so complicated a problem in a systematic quantitative way it has, of course, been necessary to make some restrictive assumptions. Thus, our "model" of (a part of) the logistics system (like any model of anything), is only an approximation of reality. Therefore, it should not be used as a tool in making decisions until some of the elements of reality left out have been incorporated or until it is determined that they do not affect the

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[1] Berman, E. B., Clark, A. J., <u>An Optimal Inventory Policy for a Military Organization</u>. The RAND Corporation Paper P-647, March 30, 1955.

results too seriously. Many institutional factors have been ignored. No account has been taken of the pressures of funding, budgeting and accounting practices; of the administrative costs of making small as compared with large contracts, or of other problems of contract negotiation and administration, or of the economies of large scale production or batch lot production, nor is any account taken of the possibility of contract cancellation.

The technique deals with each part separately, and it is intended that if it were used operationally, the computations for each part would be made many times during the life of the applicable airplane. Each computation would indicate for a relatively long period into the future the amounts to be procured, the time at which they should be delivered, the amounts and timing of repair and whether repair and procurement should be routine or expedited. Although the technique would provide a tentative schedule extending far into the future, it would be desirable to postpone binding commitments on the latter portions of the program to take account of new developments and perhaps to await a later recomputation.

The approach taken in this study has been to define the objective, or task, of the logistics system to be used in making decisions concerning aircraft spare parts requirements, and then to select a criterion that will choose the "optimal" or best system for achieving the objective. The objective of the logistics system used is: to produce a stipulated average number of mission-ready aircraft, which are capable of performing the desired flying activity for peacetime training and wartime operations. The criterion used to choose between logistics systems is cost. That system which would meet the given objective at minimum total cost, suitably defined, is considered best.

The interrelationships involved are complex. The changing of one segment has an effect on the other parts of the system. Thus, reduction in repair cycle time or resupply time may reduce the procurement requirement, but increase the costs of repair or resupply. Similarly, a reduction in the procurement lead time may reduce the requirements for procurement and repair, but the purchase price may be greater. Consideration of such trade-offs is important in understanding the logistics system and in comparing alternative logistics policies. At the same time the logistics system can (and does) trade-off logistics system cost against its effectiveness. The greater the number of stock shortages a system has the more it requires of priority action and of aircraft to compensate for those out of commission awaiting parts, if it is to achieve the given objective of mission-ready aircraft and flying hour capability. Both of these results can be approximately measured in dollar terms as follows: we can estimate the cost of priority action directly, and we can measure the cost of AOCP's by estimating the added cost of aircraft which would have to be brought into the system to offset the higher AOCP rates.

Thus, the alternative logistics systems (or sets of policies) range from those that spend comparatively little on stocks and delivery or repair capability, but have high AOCP rates, to those that spend relatively large amounts on the logistics system proper, and have lower AOCP rates. We want to choose that system that can minimize the total overall cost, including the extra expenditure for aircraft to cover the AOCP's. The technique described in this paper presents one way of trying to accomplish this choice under the assumptions used.

The technique can take account of probable deviations of actual demand from expected demand, which other RAND studies have shown to be important in logistics planning.[1] The study also discusses the possibility of deferring procurement decisions, and describes a possible technique for measuring in dollar terms the value of deferring a procurement decision.

This is the first of a series of studies defining the factors involved in making the procurement and repair decisions. As yet, the technique is untested. Therefore, it is subject to change and modification. It is our plan to test the technique using realistic data, and, if practical, to compare its results with experience. If it appears satisfactory, we shall use it in studies of the characteristics of a good logistics system. These studies may also be useful in the development of procedures for computing requirements in the Electrologs Project being conducted by the Air Force at Oklahoma City Air Materiel Area, with RAND technical assistance.[2] All the data needed for this technique are not now available, but with considerable research can probably be developed from Air Force sources.

To summarize: the significance of this study lies in providing an objective and quantitative statement of some of the central elements in procurement and repair decision making, and in providing a preliminary technique for reaching decisions systematically on the basis of these relationships. It is expected that it will be possible to develop further or revise the technique and to obtain the necessary data sources to produce a useful logistics tool.

M. A. Geisler

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1 Brown, B. B., Geisler, M. A., Analysis of the Demand Patterns for B-47 Airframe Parts at Air Base Level, The RAND Corporation Research Memorandum RM-1297, July 27, 1954.

2 McNeill, R. B., Berman, E. B., Clark, A. J., Nelson, H. W., A Proposal for a New Air Force Supply Procedure, The RAND Corporation Research Memorandum RM-1417, January 24, 1955.

A MODEL OF THE PROCUREMENT-REPAIR DECISION FOR A SPARE ITEM

Edward B. Berman

SUMMARY

This study presents a technique for computing the procurement and depot repair requirements of individual aircraft spare items for an aircraft program so that the expected total cost of procuring, repairing, and holding the stock, plus the expected cost to the logistics system of spare parts depletions, approximates the minimum.

The technique allows for the variability of demand both predictable and unpredictable, and chooses between expedited and routine procurement, and expedited and routine repair, on the basis of expected minimum total cost. It also determines how the total cost should be distributed as between logistics support (which includes the procuring, repairing, distributing, holding, and other costs associated with spare parts), and investment in, and support of, a pool of aircraft awaiting spare parts.

The method will be used to select procurement, repair, and distribution policies for aircraft spare parts from alternatives which include different repair cycles, procurement lead times, and internal resupply times, as well as the possibility of changing these from one phase of an aircraft's life to the next.

The method will also be evaluated as a possible technique for use by the Air Force in requirements calculations and repair scheduling. For this purpose, we shall supplement it with a device for determining when it is advantageous to defer the procurement of spare parts, because of the high obsolescence risk in the early period of an aircraft's life.

A MODEL OF THE PROCUREMENT-REPAIR DECISION FOR A SPARE ITEM

Part I

INTRODUCTION

The following represents an attempt to apply a mathematical model[1] as a tool in guiding Air Force decisions in procurement, distribution, and repair scheduling for spare parts used on aircraft. The model attempts to provide a technique for reaching a decision in these areas based on achieving a given combat capability with the further constraint that the decision reached should be the one consistent with the least cost to the Air Force as a whole. The model is designed to recognize the fact that a decision to procure too little of a spare part will cause the Air Force to incure excessive depletion costs; and that a decision to procure too much of a spare part will incur too large an investment in the spare, and excessive holding costs (interest, warehousing costs, fair wear and tear depreciation, obsolescence, and so forth). The model is designed also to choose between expedited and routine repair on the basis of cost considerations in which the cost of expediting repair is compared with the savings obtainable in the form of reduced procurement requirements.

The technique provides a procurement and repair plan for the remaining life of the spare item. In using the technique, however, this plan would be used to determine a set of decisions for only the immediate future; some time later, a new computation would be performed, to obtain a procurement and repair plan for the life of the spare item then remaining, which plan would again determine a set of decisions for the immediate future. In short,

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1 The model employs linear programming techniques.

the technique implies a set of successive recomputations, each one of which provides a lifetime plan. It is the expected cost of the lifetime plan which the technique will minimize.

We shall use this technique for two purposes. First, we plan to use it as a device for testing alternative procurement, repair, distribution and transportation policies, with the hope that some general policy recommendations may result. In addition, we shall test the technique for its usefulness to the Air Force as a substitute for present methods of calculating requirements and establishing repair schedules, particularly for high cost spare parts.

Section I, Decisions, provides a discussion of the kinds of decisions which the technique will reach. These decisions are the outputs of the technique. Section II, The Decision Environment, discusses the background against which these decisions are, and will be obtained. This background represents the conditions under which the Air Force now operates. Section III, Objective, presents the criteria used by the technique in forming its decisions. Section IV, Structural Elements of System, Section V, Cost Elements in System, and Section VI, Cost Elements Not in System, together present the Air Force world which the technique assumes to exist. Section VII, Interactions, describes the ways in which the elements of the assumed world, described in Sections IV through VI, influence the decisions described in Section I. Section VIII, Method in Brief, describes the technique in general non-mathematical terms. The detailed description of the technique is deferred to Part II.

I. DECISIONS

The model is designed to provide guidance in the decisions of procurement (how much do we buy and when), repair, (how much, and how fast do we repair), and distribution, (where do we place the serviceable items that are available and when), for one recoverable spare item applicable to one model or series of aircraft or other end-item. The technique has been specifically designed for the recoverable item with a high unit cost, and a low and erratic demand - the kind of item which represents a large fraction of the Air Force dollar investment in spare parts inventory.

II. THE DECISION ENVIRONMENT

In designing a technique to assist in guiding the decisions mentioned above, we were forced to recognize the environment in which these decisions will be made. In this section, we shall consider several elements of the Air Force decision environment. These elements are not necessarily the same as the ones assumed by the technique we shall describe. We defer to Sections IV and V a consideration of these latter elements.

LONG PROCUREMENT LEAD TIME

The procurement lead time for many aircraft spare items is extremely long; in the order of years. In addition, the Air Force "rules" for initial provisioning are such that often much of the initial provisioning of an aircraft model or series is determined (and the Air Force is committed thereon) before the first of the airplanes is accepted by the Air Force.

UNCERTAINTY OF DEMAND

The Air Force suffers from an unusually high uncertainty of demand for its spare items. The extent of this uncertainty results from several factors, among which are:

1. The fact that demand for most of the spare items is very low.

2. The continual changes in configuration of aircraft in production lead to a large variety of spare items. Frequently these items are applicable to only a few aircraft in a major type; and, as a result, there are major differences among different blocks of a type of aircraft, for example the B-47.

3. As the aircraft are modified (i.e., the configuration of the older planes are brought up to those of the newer planes) many of the spare items applicable to earlier configurations become obsolete.

The characteristically low demand (75 per cent of the Air Force active spare items have a worldwide demand of 10 or less a year)[1] leads directly to a high variation in demand relative to the average demand. An item with an expected demand of 1 next year is likely to incur a demand of 2, whereas an item with an expected demand of 1,000 is much less likely to incur a demand of 2,000.

The high probability of obsolescence for spare items implies a high probability that many items currently in the Air Force inventory will have no demand in the relatively near future.

HIGH OBSOLESCENCE RISK

As noted above, continual changes in the configuration of aircraft cause many of the items applicable to early configurations to become obsolete. This high incidence of spares obsolescence not only contributes to the uncertainty of demand, but it also should influence the decision to procure spares, tending both to postpone and to reduce the amount of procurement.

ALTERNATE REPAIR CYCLES

The Air Force has considerable choice in the length of the repair cycle for spare items. It can choose to use either surface or air transportation to move reparables from base to depot and to move serviceables back to the base. It can choose to allow reparables to accumulate in the reparable warehouse to gain the economies of production-line repair, while lengthening the repair cycle, or it can forego these economies by using job-lot repair, to shorten the repair cycle.

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[1] Geisler, M. A., Mirkovich, A. R., Analysis of Worldwide Data on Aircraft Spare Parts As to Unit Cost, Quantity and Value Issued, and Inventory Value, The RAND Corporation, Research Memorandum RM-1481, May 6, 1955.

ALTERNATE PROCUREMENT LEAD TIMES

The Air Force has a choice between the use of expedited procurement which, at some cost, will shorten the procurement lead time; and routine procurement in which savings in administrative expense, and perhaps in the price paid for the spare items, must be offset against the loss in time and flexibility resulting from a longer procurement lead time.

ALTERNATE DISTRIBUTION POLICIES

The Air Force can choose to stock its spare parts primarily in depots; to concentrate them in a few bases; or to spread them among many bases. Each of these policies has different implications on procurement.

III. OBJECTIVE

Basically, there are two criteria that must be considered in making any spare parts decision: the cost implications, and the military effectiveness implications of the decision. The cost implications are represented by the procurement, storage, transportation, repair, and administrative costs of buying, repairing, or distributing the spare parts. The military effectiveness implications are represented by the number of aircraft which may be expected to be mission-ready and capable of flying a specified number of flying hours or sorties as a result of the decisions. In the procurement-repair technique, we avoid the difficulties which result from a dual criterion by taking the level of military effectiveness as given and determining those policies that will permit us to achieve that level of effectiveness at least cost.

We assume that active aircraft will be in one of four statuses at any moment of time: (1) They may be in-commission or mission-ready; (2) they may be AOCP (aircraft out of commission for parts)[1]; (3) they may be AOCM (aircraft out of commission for maintenance); (4) or they may be undergoing depot maintenance. In using the technique we assume that we are given the number of aircraft which the Air Force programs for the in-commission, the AOCM, and the depot maintenance categories, and that we may add to these the number of aircraft which we expect to be AOCP as a result of our logistics policies. We also assume that the Air Force has a given program of aircraft activity which will determine the amount of demand for spares. The specified

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1 We define the AOCP to include all time lost from an in-commission status which results from a failure of supply to meet demand for a spare item, including all work stoppages and work slow-downs in maintenance which may be attributed to that failure.

program for in-commission aircraft, together with the projected program of aircraft activity, represents the fixed military effectiveness which is taken as given. The number of aircraft in AOCM and in depot maintenance status are considered given because the policies determined by the procurement-repair technique do not affect those quantities.

We can then assume that the logistician may choose to achieve the in-commission and flying hour objectives by buying very few additional aircraft to cover expected AOCP's, and investing heavily in logistical support[1] in order to keep the AOCP rate very low; or alternatively, he may buy a larger number of additional aircraft to cover expected AOCP's, and, since he could then endure a high AOCP rate, invest relatively little in logistical support. Since military effectiveness is assumed to be identical under either plan (or under intermediate plans) we assume that the logistician may form his decision solely on the basis of minimizing the total cost of the additional aircraft plus logistical support.

The objective of the proposed technique then is to determine the amount of logistical support, and the amount of whole aircraft purchased to cover expected AOCP's that will achieve the specified in-commission aircraft and flying hour requirements at lowest expected cost. In order to achieve this objective, we charge a depletion penalty against the logistics system for each day in which an aircraft is out of commission for parts, equal to the purchase and carrying costs of the aircraft divided by the number of days in its expected useful life. The AOCP charge,[2] if it were accumulated over the

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[1] We shall use the term <u>logistical support</u> to include the system that performs the procuring, holding, transporting, repairing, handling, and administering of spare items.

[2] The AOCP charge is not the whole depletion penalty. The depletion penalty would also include the extra administration, transportation, and communications costs of depletion. See Section II of the Mathematical Appendix.

whole life of the aircraft, and over all applicable spare items, should be just sufficient to pay for the additional aircraft bought to compensate for expected AOCP's.[1]

[1] In dividing the purchase and carrying cost of the aircraft by the number of days in its expected useful life, we implicitly assume that the additional aircraft purchased to cover expected AOCP's will, in fact, cover the actual AOCP's only on the average. There will be a tendency for the actual number of AOCP's to exceed the number of additional aircraft at some periods, and to fall short in other periods. We feel that an average coverage is sufficient, however, because the AOCP rate tends to be insignificant when compared to the AOCM rate, and thus the slight variation in the actual AOCP rate around its average will be swamped by the variation of the actual AOCM rate around the expected AOCM rate. There will, however, be a tendency for the actual AOCP rate to exceed the average expected AOCP rate during the period of maximum inventory and activity of the aircraft. If it appears that the AOCP rate, in any application of the technique, will be a significant per cent of the combined AOCP and AOCM rate, we may adjust the definition of the AOCP charge by dividing the cost of the aircraft by the number of days in the peak of inventory and activity, rather than in the useful life of the aircraft. In this way, the technique will purchase enough additional aircraft to cover the expected AOCP rate when that rate is at its highest.

IV. STRUCTURAL ELEMENTS OF SYSTEM

In this section, we shall describe some of the more important structural elements of the system which the technique assumes. Let it be clearly understood that these structural elements only approximate the real world of the Air Force, as discussed above in Section II. To the extent that they do, the technique may be a worthwhile device for making decisions.

CHANGING DEMAND OVER TIME

We assume that demand for the spare item is related to the number of aircraft in the inventory and their activity, although perhaps with a time lag; and that, typically, demand for a spare item rises during the phase-in of the aircraft, attains a comparatively high level during its prime of life, and falls during the phase-out of the aircraft. Consequently, the technique has been designed to deal with spares demand which is either stable or changing over time.

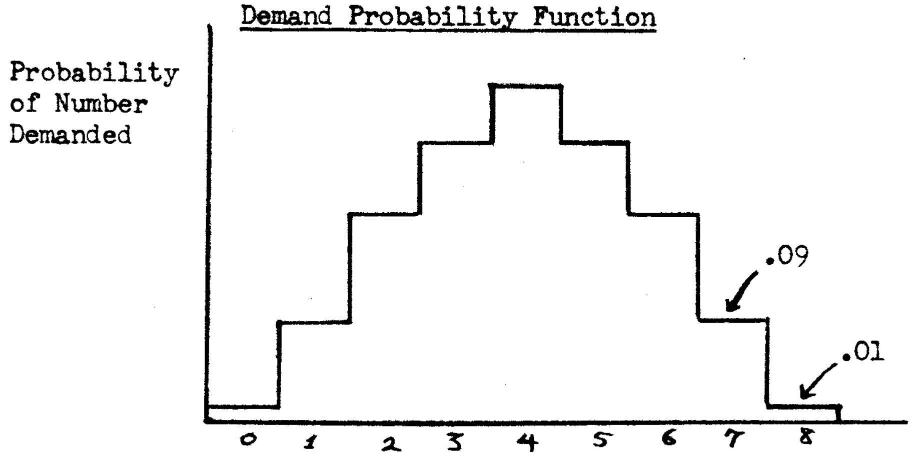
USE OF A PROBABILITY FUNCTION TO REPRESENT DEMAND

The model uses probability functions exclusively to represent demand for the spare item within each time period, both at a base and for the system as a whole. The alternative to using the probability function is to use the average demand as the true demand. Because the demand for aircraft spare items is so erratic, the use of expected demand may be inadequate.

The importance of using a probability function instead of an expected demand may be illustrated through the use of a simplified stockage example, in which we may imagine that there is one base to be stocked for one month with one spare item. Let us assume that a depletion costs $10,000 for the month and that the demand function is given as shown in Figure 1.

Figure 1

Demand Probability Function



Number Demanded

Now, let us suppose that we are using an expected demand and that our "rule" tells us to order the expected demand plus a 50 per cent safety factor. We should therefore order the expected demand, 4, plus a safety factor of 2, or 6 in total. Using the probability function, we note that there is a one-tenth probability of a demand of 7 or more, and a one-hundredth probability of a demand of 8. Thus, if the unit cost of the item were $800, it would pay us to place 7 items at the base, since the seventh item would have a one-tenth probability of being needed, or in other words, a one-tenth probability of saving us a $10,000 depletion penalty, and thus, an expected saving of $1,000, as compared with a cost of $800. Similarly, if the unit cost of the item were $50, it would pay us to put 8 items at the base, since the eighth item has a one-hundredth probability of saving $10,000 or an expected saving of $100, as compared to a cost of $50. We are assuming in this simplified model that the items have no use if there is no demand for them at the one base in the one period.

The probability function certainly has given us a different solution than the one obtained by use of the "rule." Its most important aspect, however, is its ability to differentiate policy according to differing cost considerations. In using the expected demand "rule," there was no easy way to differentiate our policy according to the unit cost of the item. Through the use of the probability function, this differentiation became automatic.

The inability of an arbitrary rule to differentiate its policy implies that the "rule" will always give an incorrect result for some items. If, for example, the "rule" were to buy the expected demand plus 75 per cent, it would indicate a stockage of 7 for both the $800 item and the $50 item. This result would be correct for the $800 item, but incorrect for the $50 item. If, on the other hand, the "rule" were to buy the expected demand plus 100 per cent, it would indicate a stockage of 8 for both items, which would be correct for the $50 item, but incorrect for the $800 item.

USE OF PAST DEMAND TO DETERMINE REPAIR

In the model, we have treated the availability of serviceable spare items from repair as a function of demand in previous periods; repair availability is thus an element generated internally by the model.

In the real world of the Air Force, the supply of serviceable spare items from repair is usually limited by the supply of reparable carcasses. We have therefore assumed that the supply of reparable carcasses is the determining factor as to the number of overhauls that can be performed, if necessary. This implies that the equipment, manpower, and bits and pieces will normally be available to perform the overhaul. This seems reasonable in view of the expansibility of repair capital capacity through employment of private industry on contract.

The supply of reparable carcasses, in turn, is strictly a function of demand in previous periods. Demand for a spare item always occurs in the form of an exchange of a serviceable spare for a non-serviceable spare. In the case of a recoverable-type spare, there is always an item received by the base in exchange for the item issued. The item received in exchange may be declared either condemned (not worth repairing) or reparable (worth repairing). The reparable spares form the input of carcasses into the overhaul shop. In this model, we do not treat base repair separately, but lump items in base repair and serviceable items together, since the base repair usually is relatively short.

THE PERIOD OF DELIVERY

The model determines the amount to be procured and the periods in which items should be delivered.

DISTRIBUTION

In the Air Force, the aircraft may be distributed (and therefore demand for the spare item may exist) at many or few bases. The procurement requirement may or may not be larger under an extensive dispersion of the aircraft than if it is narrowly distributed. The model has been designed to reflect these distributional elements in its procurement-repair decisions.

THE WEAROUT FACTOR

The wearout factor is defined as the ratio of the number of a given line item condemned to the total number of the line item demanded. This ratio affects the probability of depletion, the probability of obsolescence, and the expected cost of holding the spare item. The model has been designed to recognize all of these implications.

TWO REPAIR CYCLES

In the Air Force, there are a large number of alternative repair cycles possible with any one spare item. We approximate this by assuming that there are only two cycles from which to select - an expedited repair cycle, and a routine repair cycle. The model is designed to determine repair decisions in terms of the type of cycle (routine or expedited) which should be used and the number of the items to be repaired in each period.

TWO PROCUREMENT LEAD TIMES

As in the case of the repair cycle, the system assumed by the technique contains only two procurement lead times - an expedited procurement lead time and a routine procurement lead time. To facilitate computational work the assumed system contains less variety of choice than the real world.

DEPOT BASE PIPELINES

The model assumes that premium communications and premium transportation will always be used between depots and bases to alleviate base depletions. However, in positioning materiel on bases to prevent anticipated base depletions, the Air Force might conceivably choose to use either routine (slow) or premium (fast) transportation. The model is not capable of choosing between these media, but must be told which form of transportation is to be used. It is, however, capable of determining the expected system costs under each of the two transportation assumptions; a comparison of these costs could then be used for determining which medium of transportation should be selected.

ATTRITION AND SALVAGE GAINS

The model is capable of treating two additional ways of increasing the spare item inventory: attrition and salvage. Attrition gains are increases in the system stock of serviceables and reparables resulting from the accidental destruction of an aircraft from which the item under study is recovered. <u>Salvage gains</u> are increases in the system stock of serviceables and reparables resulting from the planned dismemberment of those aircraft no longer needed in the aircraft inventory. Items entering the system as either attrition gains or salvage gains are assumed to be reparable, not serviceable.

The technique treats both attrition and salvage gains as expected values, rather than as probability functions.

V. COST ELEMENTS IN SYSTEM

In this section, we shall describe the costs which the technique considers in forming decisions.

DEPLETION PENALTIES

The model is designed to consider two kinds of depletion penalties; one, the system depletion penalty, is incurred when there is a demand for a spare item, and no serviceable spare item exists anywhere in the system; the second, the base depletion penalty, is incurred when there is a demand for a spare item, and no serviceable spare item exists at the location of the demand but an item is available elsewhere in the system. Each of these costs is a function of the amount of depletion, and of the actions available to alleviate the depletion.

We do not assume that every depletion results in an AOCP. Thus, we allow for the possibility of cannibalizing a part from an aircraft in maintenance, or the use of a next higher assembly. We vary the depletion penalty according to the number of depletions that have occurred at the base during the period, since the first few depletions may not require that an aircraft be AOCP. The depletion penalty includes not only the AOCP charge, or the cost of maintenance cannibalization, but also the extra administrative, transportation and communication costs of depletion.

THE HOLDING COST

The model recognizes a given holding cost, charged against each spare item brought into the system for each unit of time it is in the system. This holding cost conceptually includes costs of warehousing, interest, fair wear and tear depreciation, and costs of modifying the spare itself if required by the system, but it does not include obsolescence costs, which are handled separately.

OBSOLESCENCE

We may define two kinds of obsolescence as applicable to the spare item: engineering change obsolescence and terminal obsolescence.

Engineering change obsolescence occurs when the spare item becomes obsolete prematurely; that is, before the applicable aircraft becomes obsolete. Frequently, an aircraft is modified during its life in such a way as to cause one or more spare items, which were applicable to previous configurations, to become inapplicable. The technique represents this kind of obsolescence primarily by means of the demand probability functions themselves. The probability of zero issues in any period represents the sum of (1) the probability of the spare item suffering engineering change obsolescence in or before that period, and (2) the probability of the spare item remaining applicable, but zero demand occurring. The probability of a demand x where x is greater than zero is the product of the probability of x demands occurring if the item does not become obsolete, and the probability of the item not becoming obsolete.

Terminal obsolescence occurs when the spare item becomes obsolete because the aircraft has become obsolete. In valuing terminal obsolescence, we must first determine whether or not the Air Force is willing to phase out the applicable aircraft as an incomplete aircraft - that is, missing one or more spare items. If they are willing to do so, the concept of terminal obsolescence becomes meaningless, since the cost of procuring an item in that case is the same whether or not the spare item remains in the system at the end of its useful life.

If the Air Force is unwilling to phase out any of the applicable aircraft as incomplete aircraft, perhaps because of a plan to mothball them for later use, a plan to present them to a friendly nation, or because of future uncertainties, the number of items condemned becomes a fixed minimum requirement for procurement of the spare item. In this case, the technique will not charge the procurement unit cost against the logistics support decision for all items that are condemned, since it is not within the discretion of the logistics system whether that number of items is bought or not. The system is, however, charged a holding cost if the items are bought too early, and a depletion cost if the items are bought too late.

The technique has been designed to provide decisions under either of the above assumptions.

PROCUREMENT UNIT COST

The technique treats the unit cost as a constant cost per item purchased, independent of the time of purchase (or delivery) and independent of the amount of procurement. The unit cost represents the procurement cost per item, assuming routine procurement, including therein the cost of transportation to the first Air Force destination, and a fair share of the administrative costs of procurement.

EXPEDITED PROCUREMENT PENALTY

The expedited procurement penalty is a charge levied against each item procured under expedited procurement, which represents the additional cost of procuring an item under expedited procurement.[1]

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[1] The expedited procurement penalty includes additional first-destination transportation and additional administrative cost.

COST OF ROUTINE REPAIR

The cost of routine repair covers in concept all costs associated with putting an item through routine repair including the costs of (1) packing the reparable at the base, (2) transporting the reparable to the depot, (3) inspecting the reparable at base and depot, (4) handling the reparable at base and depot, (5) overhauling the item, and (6) packing and transporting the serviceable back to the base.

COST OF EXPEDITED REPAIR

The cost of expedited repair includes all of the kinds of costs described above for routine repair, except that these costs are associated with the use of the expedited repair cycle. Some of the costs may be identical under both routine and expedited repair. This cost is the total, not the additional, cost of expediting repair.

VI. COST ELEMENTS NOT IN SYSTEM

In this section we shall describe some of the more important costs which the technique does not consider in forming its decisions.

SET-UP COSTS

In the real world, there are several set-up or fixed costs which are relevant to the procurement-repair decision. For example, the manufacturer might charge $1,000 per order, plus $500 for each item ordered. If this were the case, the first item entered on an order would cost $1,500, whereas the first two would cost only $2,000. In this event, the procurement decision would undoubtedly tend to be more bunched in terms of the time-phasing of delivery than if a constant unit procurement cost were used.

Set-up costs could be encountered in several of the elements the model attempts to consider; for example, in the procurement order (as described above), in repair scheduling, and in shipments from depot to base. The model considers set-up costs only in repair scheduling, and considers them only indirectly. Fortunately, the most important set-up cost appears to be associated with the initiation of production of an item, and hence it is probably irrelevant to decisions on spare parts (since production must be set-up for the aircraft in any case).[1] For an expensive spare (and recoverable spares tend to be expensive) the set-up costs for ordering and for depot-to-base shipments seem to be negligible when compared to other costs.

VALUE OF DEFERRING DECISIONS

Because of the unpredictability of future needs, it may be advantageous

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Apparently the set-up costs associated with reinitiation of production are not too significant.

to defer decisions being considered. In spares procurement, this advantage is primarily associated with the fact that the passage of time may bring an improved estimate of the demand probability functions for the future.

The technique computes the best procurement and repair schedules for the remaining lifetime of the spare item as it appears at the time of computation. However, in using the technique, it is necessary to commit oneself on only some of these decisions at that time - those that could not be deferred without some extra cost. For example, if the technique's solution indicated that 2 items should be delivered in the 17th period, and only 4 periods were necessary to avoid the expedited procurement penalty, the Air Force could postpone the actual decision (here, placing the order) until the 13th period without adding any cost. Up to the 13th period, the Air Force could recompute the solution with improved estimates of the demand probability functions. We have, however, no way of deciding within the present technique whether or not it would pay to postpone delivery until the 18th period in order to obtain a better estimate of the demand probability.

In using the technique, the Air Force could operate under the assumption that all decisions will be deferred as long as the deferment cost nothing, but it could not with the technique determine whether or not any further deferment would be desirable. However, Part III of the Appendix presents a method for determining the value of decision postponement, using an additional procedure to supplement the technique presented here.

VII. INTERACTIONS

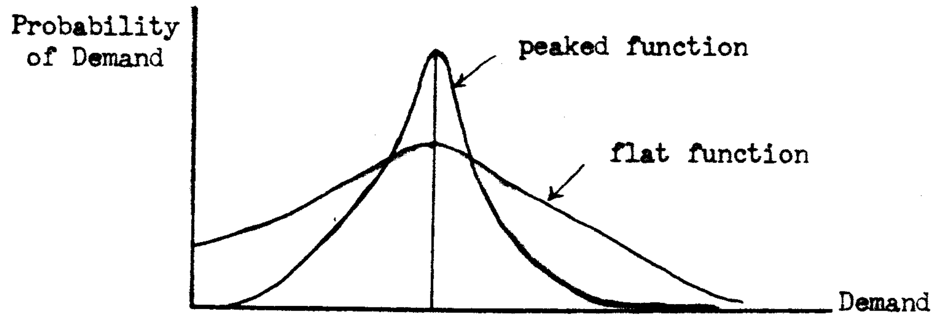
In this section, we shall describe the effect of the structural and cost elements upon the decisions reached by the technique. Let us first introduce a new term which will be used rather frequently in the following discussion, namely, requirements. An increase in requirements will imply that there is a tendency for the amount procured, the use of expedited repair, and the probable amount of uncovered depletion all to increase. One or more of the three will increase as a result of increased requirements, but there will be a tendency for all to increase. Only the discontinuity implicit in having to buy whole items, rather than fractions of items, will prevent all three of these effects from occurring in fact. A decrease in requirements will have an exactly contrary meaning.

DEMAND

We have discussed the effect of using a probability function. We now consider the effects of various shapes of such functions. It is obvious that a greater amount of demand per time period will tend to increase requirements. There are less obvious interplays, however, between the shape of the demand probability function in a given time period, the changes in the demand function over time, and the procurement-repair decisions.

If demand probability in a period is fairly well concentrated, or in other words, if there is a peaked demand probability function, the requirements will probably be smaller than if the demand probability function is flat, given an equal mean demand in both cases. This is because a more peaked demand means that the mean demand is much more representative, and the system is less subject to extremely high demands than is the case with a flat demand curve.

Figure 2

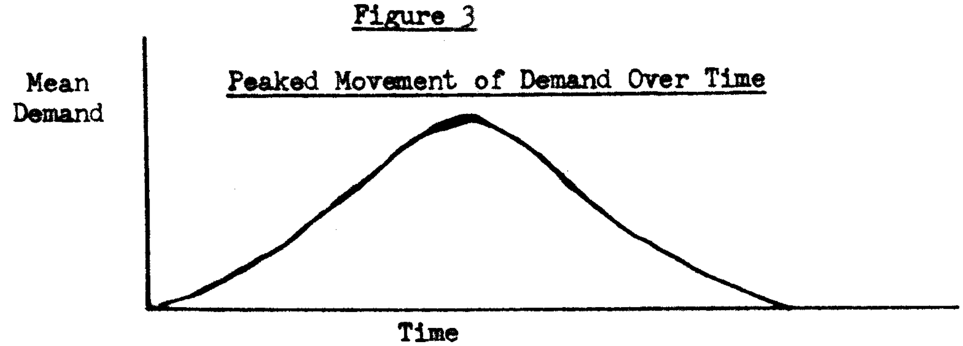


Peaked and Flat Demand Functions

The preferred procurement plan will usually involve more procurement than the mean demand in most periods. Under such conditions, there will be more uncovered probability of depletion (demand in excess of stock on hand) with the flat demand function than with the peaked demand function. This means that more stock will be carried for items with flat demand functions to avoid depletions.

A somewhat more complex set of interrelationships may be expected from the change of total system demand over time, which has important impacts on the relative use of procurement and expedited repair. If the mean system demand over time is peaked as in Figure 3 there will be a tendency to substitute expedited repair for procurement, since the use of expedited repair in but a few periods will suffice to reduce procurement by a substantial amount.

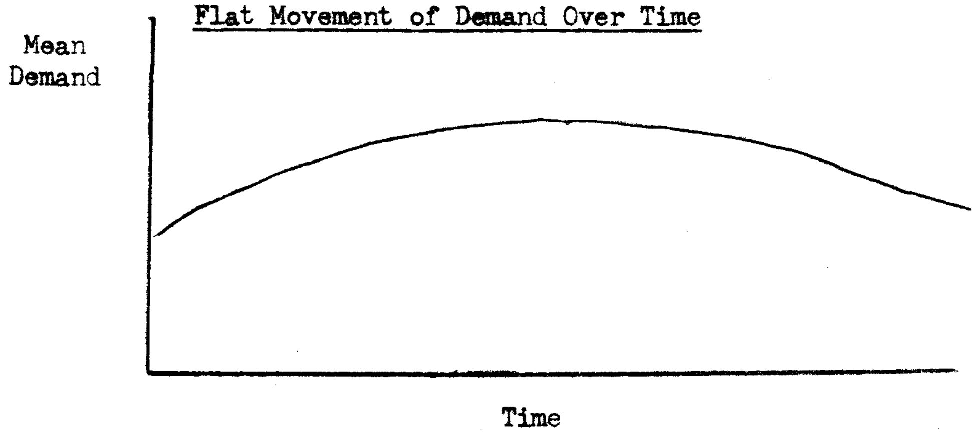
Figure 3



If, on the other hand, the movement of mean demand over time is flat, as in Figure 4, there will be a tendency to procure heavily and to reduce the use of expedited repair, since it will be necessary to expedite repair in many periods in order to obtain the benefits of reduced procurement.

Figure 4

Flat Movement of Demand Over Time



In the case of a peaked movement of demand over time, there will also be a tendency to allow a larger amount of uncovered depletion probability, (or in other words, to permit a larger AOCP charge), since peaked mean demand over time implies that the cost of reducing the uncovered probability of depletion is high. In other words, relatively more items are required to cover all depletion probability in the case of peaked mean demand over time. Thus, each item procured will contribute less to preventing depletions under peaked movement over time than would be the case under flat movement over time. In the preferred procurement plan, therefore, we should expect less procurement and more uncovered depletions for peaked demand over time.

REMAINING LIFE OF SPARE ITEM

The remaining life of the spare item[1] has an important impact on the

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1 Represented by the number of periods in the model.

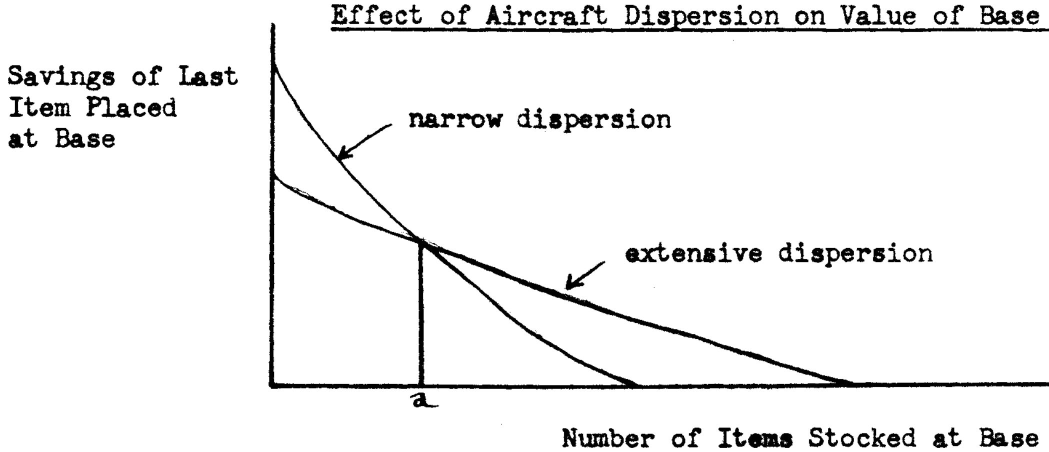
choice between procurement and expedited repair. If the remaining life is long, there will be a tendency to procure more, and to expedite repair less often. If the remaining life is short, the opposite tendency will prevail. This interrelationship between remaining life of the spare item and the choice between procurement and expedited repair is a consequence of the natures of these alternatives. Procurement is an investment. Once an item is procured, an asset is created which will have potential use throughout the lifetime of the spare item. On the other hand, the cost of expediting repair is an operating expense; the benefits which result from incurring this expense are comparatively short-lived. Thus, a long remaining lifetime will tend to raise the payoff of a shift from operating expense to investment, or from expediting repair to procurement.

DISPERSION OF AIRCRAFT

The impact of aircraft dispersion on requirements will vary depending on circumstances. There will be some cases in which an extensive dispersion of aircraft, and thus a large number of bases at which demand may occur, will tend to reduce requirements, and other cases in which an extensive dispersion will have the opposite effect.

Figure 5

Effect of Aircraft Dispersion on Value of Base Stockage



Notice that in Figure 5 the narrow dispersion offers higher savings from the first few items placed at base level. This results from the fact that the Air Force can predict quite well where demand for the spare item will occur, given a narrow dispersion; the first few items placed at bases will have a fairly high probability of saving money by preventing base depletions. With an extensive dispersion, there will be a small amount of demand at many bases. Thus, the first few items placed at base level will have a relatively low probability of saving money by preventing base depletions.

Notice further than the extensive dispersion offers higher incremental savings at base level where the stockage at bases is very large. This is a result of the relatively low demand at each base which arises from the extensive dispersion, and which causes in turn a greater degree of demand instability. There is thus a larger amount of base level depletion probability to cover under an extensive dispersion than under a narrow dispersion.

Since the two curves necessarily cross each other (at a in Figure 5), we may say that narrow dispersion will tend to raise requirements if the procurement decision implies base stockage less than the crossing point, a; and that narrow dispersion will tend to reduce requirements if the procurement decision implies base stockage greater than a.

WEAROUT FACTOR

A high wearout factor will tend to increase requirements for several reasons. First, a high wearout factor implies that, given the amount procured, there will be fewer items left uncondemned; and therefore each item left will have a higher probability of saving money by preventing depletions. Secondly, the high wearout factor implies that a given item will have less probability of incurring terminal obsolescence since there will be fewer items left over at the end of the program. This is equivalent to a reduction in the procurement price, and it therefore tends to increase the preferred amount of procurement. Third, since there will be more condemnations, there will be less probability of the marginal, or additional, item incurring a holding cost in later periods of the life of the aircraft type, which also tends to encourage procurement. Thus, the general effect of high wearout rates is to increase requirements, and to encourage meeting these requirements by the use of procurement.

LENGTH OF REPAIR CYCLES

Clearly, a relatively longer routine repair cycle will tend to encourage the use of expedited repair, and a relatively longer expedited repair cycle will tend to encourage the use of routine repair. In addition, if both repair cycles are longer, this will tend to increase requirements since procurement will be used to cover demand over the repair cycle period. In fact, if the wearout factor were zero, procurement would be used only to cover demand over the repair cycle and the depot-to-base pipeline periods.

LENGTH OF PROCUREMENT LEAD TIME

A longer procurement lead time will force decisions to be made earlier, and therefore will tend to raise requirements, since demand must be projected further into the future at the time the Air Force commits itself on procurement. The longer procurement lead time thus implies a greater degree of uncertainty, which in turn implies larger requirements.

LENGTH OF DEPOT-TO-BASE PIPELINE

A longer depot-to-base pipeline will tend to increase requirements, since procurement will be used to cover demand over this pipeline period, as was true of the repair cycle pipeline.

DEPLETION PENALTIES

Higher depletion penalties will serve to make procurement and/or expedited repair more profitable since each depletion prevented will save a larger amount of money. Thus, higher depletion penalties will tend to cause larger procurement and/or a greater use of expedited repair, and will tend to cause a reduction in the amount of uncovered depletion probability.

HOLDING COST

A higher holding cost will place a penalty on early procurement, and will therefore tend to postpone procurement to relatively later periods. In addition, a higher holding cost will tend to reduce the preferred amount of procurement and make the repeated repair of a smaller stock of items preferable.

ENGINEERING CHANGE OBSOLESCENCE

A high probability of engineering change obsolescence will tend to postpone and reduce procurement for two reasons: first, engineering change obsolescence forms an important part of the value of decision deferment; second, a high engineering change obsolescence probability is represented in the demand probability functions as a high probability of zero demand. There will be some reduction of the effect, however, because a high engineering change obsolescence probability implies a relatively low probability that the holding cost will be incurred in later periods of the model. The net effect, however, is clearly toward the postponement and reduction of procurement.

REFUND FOR CONDEMNATION

If the refund for condemnation is used in the technique, implying that the Air Force wishes to phase its aircraft out as complete aircraft, the amount procured will tend to be larger than if this refund is not used, and there will probably be less use of expedited repair. This is because the refund, in effect, reduces the cost of procuring an item. A larger refund (indicating a higher wearout factor) will cause an intensification of these tendencies.

UNIT COST

A higher unit cost, other things remaining the same, will tend both to lower procurement, and to increase the use of expedited repair, since both uncovered depletions and expedited repair become relatively less expensive alternatives to procurement.

EXPEDITED PROCUREMENT PENALTY

The expedited procurement penalty will tend to delay the delivery of items from procurement to avoid incurring this penalty. If the penalty is higher, the tendency to avoid expedited procurement will be greater.

COST OF REPAIR

The cost of routine repair and the cost of expedited repair are both important since together they determine the net cost of expediting repair. If this net cost of expediting repair is high, the technique will tend to substitute procurement for expedited repair. If this net cost is low, the technique will tend to substitute expedited repair for procurement.

VALUE OF DECISION DEFERMENT

If the value of decision deferment is high, it will clearly tend to postpone procurement. A high value of decision deferment will also tend to reduce the amount procured, since the postponement of procurement will never increase the total amount procured, but may on occasion reduce it; (for example, if procurement is deferred, and, before the next recomputation, the item incurs engineering change obsolescence).

NOTE ON INTERACTIONS

The interactions between elements and decision which are described above represent the way the interactions will occur in the proposed technique. We believe, however, that they also represent the way a manager would tend to act if he had the necessary information available. The technique will reach a decision which weighs the several counteracting factors so that the total cost over the remaining period of the program is minimized.

PART II

M A T H E M A T I C A L

A P E N D I X

VIII. METHOD IN BRIEF

For reasons of computational convenience, we do not determine all of the procurement-repair decisions simultaneously. Instead, we proceed to the preferred procurement-repair decision as follows: we first determine a range within which the preferred procurement decision will lie by sub-optimizing[1] on procurement, given the length of the repair cycle; we then determine the preferred procurement-repair plan by sub-optimizing on repair, given procurement, for each procurement decision within the range.

The first step is called the "routine repair sub-optimization." In this step, we assume the use of routine repair in all periods, and solve for the preferred procurement plan (or schedule of deliveries from procurement by period) under that assumption. The inputs to the solution are the structural and cost elements, with the structural elements constrained to the use of routine repair in all periods. The output is that procurement plan which minimizes total system cost under the assumption of routine repair in all periods. The solution given by this step represents the maximum amount of procurement - no combination of routine and expedited repair would require more - and the delivery period of each item procured (i.e., the 1st, 2nd, 3rd, etc., item procured) will represent the earliest period in which that item should be delivered under any combination of expedited and routine repair.

The second step is called the "expedited repair sub-optimization." In this step, we assume the use of expedited repair in all periods, and solve

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1 The term sub-optimization is used in this paper to represent a solution of a problem reached under a set of constraints, which may not be constraints in the real world.

for the preferred procurement plan under that assumption. The inputs to the solution are the structural and cost elements, with the structural elements constrained to the use of expedited repair in all periods. The output is the preferred procurement plan under the assumption of expedited repair in all periods. The solution given by the expedited repair sub-optimization represents the minimum amount of procurement - no combination of routine and expedited repair would require less - and the delivery period of each item procured will represent the latest period in which that item should be delivered under any combination of expedited and routine repair.

The third step is called the "reverse sub-optimization." In this step, we assume a given procurement plan, and solve for the preferred repair plan under that assumption. The inputs to the solution are the given procurement plan, the cost elements, and the structural elements; the output is the preferred repair plan giving the schedule of routine and expedited repair by period. The reverse sub-optimization is performed for each procurement plan between the maximum and minimum, and a system cost derived for each. The reverse sub-optimization having lowest system cost then represents the optimal procurement-repair plan.

This procedure may be considerably shortened by applying the reverse sub-optimization to procurement amounts that are spaced apart, but lie between the maximum and minimum procurements, in order first to localize the range of procurement within which the optimal solution possibly lies. This short-cut is not necessary or particularly useful for the very low demand items for which the technique was primarily designed, since for these items the range in procurement under the two repair alternatives is probably small.

I

THE STRUCTURE OF THE MODELS

For the expedited repair and routine repair sub-optimizations, model 1 is used.

Figure 1

MODEL 1

Column Total	A	1	1	1	1	1	1	1	
	P	U_1	U_2	U_3	U_4	U_5	U_6	U_7	Row Totals
1	P_1	U_{11}	U_{12}	U_{13}	U_{14}	U_{15}	U_{16}	U_{17}	0
2	P_2	U_{21}	U_{22}	U_{23}	U_{24}	U_{25}	U_{26}	U_{27}	0
3	P_3	U_{31}	U_{32}	U_{33}	U_{34}	U_{35}	U_{36}	U_{37}	0
4	P_4	U_{41}	U_{42}	U_{43}	U_{44}	U_{45}	U_{46}	U_{47}	0
5	P_5	U_{51}	U_{52}	U_{53}	U_{54}	U_{55}	U_{56}	U_{57}	0
6	P_6	U_{61}	U_{62}	U_{63}	U_{64}	U_{65}	U_{66}	U_{67}	0
7	P_7	U_{71}	U_{72}	U_{73}	U_{74}	U_{75}	U_{76}	U_{77}	0
8(s)	P_8	U_{81}	U_{82}	U_{83}	U_{84}	U_{85}	U_{86}	U_{87}	

$$\text{Criterion: } \sum_{i=1}^{n} P_i K_{P_i} + \sum_{i=1}^{n} \sum_{j=1}^{m} U_{ij} \; K_{U_{ij}} = \text{MIN.}$$

where K_{P_i} is the cost of activity P_i;

$K_{U_{ij}}$ is the cost of activity U_{ij};

n is the number of periods in the study, including surplus;

and m is the number of U columns.

In model 1, the rows represent the time period of delivery of the items procured. The P column represents procurement, where P_1 is the amount procured for delivery in the first period, etc. The 8th period is surplus so that P_8 is the amount not procured.

If $U_{ij} = 1$, the j-th item is procured for delivery in the i-th period; U_{ij} equals either 1 or 0. Thus, column U_2 represents the second item bought, and if U_{32} is 1, the second item is bought for delivery in the third period.

We may write the following equations for this model:

1. $$\sum_{i=1}^{n} P_i = A \text{ (one equation)}$$

where n is the number of periods, and A is the maximum amount of procurement. If there is no relevant maximum amount A, one may be invented equal to or larger than the number of U columns; indicating that there is no relevant limit on the amount of procurement. A, of course, must be an integer.

2. $$\sum_{i=1}^{n} U_{ij} = 1; \quad j \text{ goes from } 1 \text{ to } m \quad (m \text{ equations})$$

where m is the number of U columns.

These equations prevent the model from buying the first item more than once, etc. We may note at this time that the model will operate correctly only if in any row, the savings from the first item bought are greater than the saving from the second item bought; the savings from the second item bought are greater than the third item bought, etc. Therefore, the model would like to consider each item the first item bought; only equations (2) prevent this.

3. $$P_i - \sum_{j=1}^{m} U_{ij} = 0; \quad i \text{ goes from } 1 \text{ to } n-1 \quad (n-1 \text{ equations})$$

Equations (3) insure that for each unit of activity entered in the

P column, there is a compensating unit of activity in a U column. The activity in the P column incurs a positive cost; the activity in the U column provides a saving (or negative cost).

Equations (1) and (2) may be called the column balance equations, and equations (3) may be called the row balance equations. Note that in total there are always $n + m$ equations in model 1.

There is a cost associated with each activity. These costs are always positive or zero in the case of a P activity, and always negative or zero in the case of a U activity. These costs are derived from the structural and cost elements described in Part I.[1/] We shall describe in detail how the P and U costs are derived from these elements in the next chapter.[2/]

The reverse sub-optimization is accomplished through the use of model 2.

Figure 2

MODEL 2

	R_1	R_{10}	R_2	R_{21}	R_{20}	S_2	R_3	R_{32}	R_{30}	S_3	R_4	R_{43}	R_{40}	S_4	Total
1	1	1													1
2			1	1	1										1
3							1	1	1						1
4											1	1	1		1
5	1		1			1									1
6			1	1			1			1					1
7							1	1			1			1	1

1/ Part I, sections IV and V

2/ Model 1 is a "transportation" type linear programming model. The "transportation" type linear programming model requires that all coefficients be either 1 or -1; that all the constants on the right-hand side of the equations be integers (these are the row and column totals); and that the model be capable of being expressed in a format such that the right-hand sides are all row or column totals. The "transportation" type linear programming model is convenient for a procurement decision since it necessarily provides triangularity and integral activity levels.

Criterion: $\sum_{i=1}^{n-1} R_k K_{R_k} + \sum_{i=1}^{n-1} R_{k,k-1} K_{R_{k,k-1}} = \text{MIN.}$

where K_{R_k} is the cost of activity R_k;

$K_{R_{k,k-1}}$ is the cost of activity $R_{k,k-1}$;

and n is the number of periods (not including surplus) in the study.

Model 2 is shown here in the detached coefficient form. Here, each row is an equation. Each cell contains the coefficient of the activity listed as the column head in the row equation. The total column represents the right hand sides of the equations. For example, equation 2 may be read as follows:

$$R_2 + R_{21} + R_{20} = 1$$

and equation 6 may be read as:

$$R_2 + R_{21} + R_3 + S_3 = 0.$$

In model 2,

R_k is equal to either 0 or 1; if it is equal to 1, repair is expedited on all reparables generated in the k-th period, when repair has not been similarly expedited in the preceding period;

$R_{k,k-1}$ is equal to either 0 or 1; if it is equal to 1, repair is expedited on all reparables generated in the k-th period when repair has been similarly expedited in the preceding period;

$R_{k,0}$ is equal to either 0 or 1; if it is equal to 1, repair is not expedited on reparables generated in the k-th period;

S_k is a surplus activity;

The particular example of model 2 shown here represents a 5 active period model, plus a 6th surplus period; a 1 period expedited repair cycle, and a 3 period routine repair cycle.

The general form of model 2, assuming a one period expedited repair cycle and a three period routine repair cycle, but not constraining the number of periods in the model, may be written as follows:

4. $R_k + R_{k,k-1} + R_{k,0} = 1$; k goes from 1 to $n-1$ ($n-1$ equations)

where n is the number of periods (excluding the surplus period) in the model. Equations (4) force the model to choose only one form of repair for any one period.

5. $R_k + R_{k,k-1} + R_{k+1} + S_{k+1} = 1$; k goes from 1 to $n-2$ ($n-2$ equations)

Equations (5) force the model to abstain from using the R_k form of expediting repair in period $k+1$, if repair has been expedited in period k.

In model 2, all $R_{k,0}$ and S_k costs are zero. The next chapter will explain in detail how the costs of R_k and $R_{k,k-1}$ are derived from the structural and cost elements described in part I. 1/

Model 2 will always contain $2n - 3$ equations in the case of a one period expedited repair cycle and a three period routine repair cycle.

1/ Model 2 is triangular for any feasible basis (this has been proven, although the proof will not be presented here), has only 1 or 0 coefficients, and has only integral right-hand side constants; we are, therefore, assured that the optimal solution will contain only integral activity levels.

II

THE DERIVATION OF COSTS

In this section, we shall describe in detail how the P and U costs in model 1, and the R costs in model 2, are derived from the structural and cost elements recognized by the technique.

Costs for Model 1.

1. The Depletion Penalty

Let us distinguish two related concepts, both of which pertain to depletion: the Stock-out and the AOCP.

A stock-out occurs when there is a demand for an item, and no item is available to fill the need. If there is no item available at the location of the demand, a base stock-out exists; if there is no item available throughout the system, a system stock-out exists.

An AOCP, aircraft out of commission for parts, occurs when a stock-out puts an aircraft, which would otherwise be useable, temporarily out of commission.[1]

The stock-out in its pure form may occur when the aircraft requiring the spare part would not have been useable if the spare part were available because it was undergoing overhaul when the need was discovered; it may occur because the aircraft was already in an AOCP status because of another spare item; or it may occur because "maintenance cannibalization" is practiced, whereby the spare item needed is removed from an aircraft entering the overhaul shop, and placed on the aircraft which is useable except for the spare.

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[1] The AOCP includes any lost time in maintenance caused by the unavailability of a spare item.

We have already indicated[1] how one might estimate the cost of the loss of utility of an aircraft in an AOCP. This cost would be additive to the stock-out cost.

The stock-out cost, in the absence of maintenance cannibalization, would consist of the extra costs of premium communications and premium transportation used to rush the spare item into the system, if there is none in the system, and to the base needing it. In the absence of maintenance cannibalization, the base depletion penalty, π_B, would be given as follows:

6. $\pi_B = s_B + a_B e_B$

where s_B is the cost of a base stock-out

e_B is the cost of a base AOCP[2]

and a_B is the ratio of base stock-outs which lead to AOCP's to total base stock-outs.

1/ Part I, section III

2/ e_B is the cost of the lost utility of the aircraft per day multiplied by the number of days in the priority depot-to-base pipeline.

In the absence of maintenance cannibalization, the system depletion penalty, π_s, would be given as follows:

7. $\pi_s = s_s + a_s e_s$

where s_s is the cost of a system stock-out

e_s is the cost of a system AOCP lasting for one period of time;

and a_s is the ratio of system stock-outs which lead to AOCP's to total system stock-outs.

If maintenance cannibalization is used, the base depletion penalty would be given as:

8. $\pi_B = s_B + b_B M$

where s_B is the cost of a base stock-out excluding the cost of maintenance cannibalization;

b_B is the ratio of base stock-outs requiring maintenance cannibalization to all base stock-outs;

and M is the cost of one exchange of the spare item; that is, the cost of installing the spare item once and removing it once.[1]

If maintenance cannibalization is used, the system depletion penalty would be given as:

9. $\pi_s = s_s + b_s cM$,

where s_s is the cost of a system stock-out, excluding the cost of maintenance cannibalization;

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[1] We assume, in defining π_B as we have, that the depot-to-base emergency pipeline is shorter than the maintenance flow-time on the aircraft, so that one extra exchange will be sufficient to eliminate the AOCP in the case of a base depletion.

b_s is the ratio of system stock-outs requiring maintenance cannibalization to total system stock-outs;

M is again the cost of one exchange of the spare item;

and c is the ratio of the number of days in the period of the time used in the model to the number of days in the maintenance flow-time of the aircraft.

The term c represents the minimum number of exchanges per period that would have to be made to prevent the demand for this item from causing an AOCP.

We shall use depletion penalties, Π_{sq} and Π_{Bq}, which will vary with the size of depletion, where Π_{sq} (or Π_{Bq}) is the cost of the q-th system (or base) depletion in the period. Each Π_{sq} or Π_{Bq} will be obtained using equations (8) through (11). In this way, we are enabled to recognize the fact that the first few depletions in a period may be covered by maintenance cannibalization, but that at some quantity of depletions, AOCP's will inevitably result.

2. The P Costs

The cost of P_i in model 1 consists of 3 elements, the procurement cost, P_{pi}; the holding cost, P_{hi}; and the expedited procurement penalty, P_{ei}.

The procurement cost, P_{pi}, is the unit cost of the spare item. Model 1 will permit the unit cost to change over time,[1] but will not permit it to change over the quantity ordered. We shall, however, assume that the unit cost is constant over both the quantity ordered and time.

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1 Perhaps because of a learning curve.

The holding cost, P_{hi}, charged as part of the cost of P_i, is based on the assumption that each item purchased will remain in the system uncondemned from the period of its delivery to the last non-surplus period of the model. A holding cost refund is included in the U costs to cover the probability of an item being condemned or becoming obsolete before that time.

10. $$P_{hi} = \sum_{k=i}^{n-1} h_k$$

where h_k = the cost of holding one item in the system in period k. We shall assume that $h_k = h_{k-1}$ for all k.

The expedited procurement penalty, P_{ei}, will be charged against the delivery of an item in any period in which the placement of an order at the time of computation of the model will not permit the use of routine procurement. In any other period, P_{ei} will be zero.

We may now define K_{P_i}, the cost of activity P_i, as follows:

11. $$K_{P_i} = P_{pi} + P_{hi} + P_{ei}$$

3. <u>The U Costs</u>

The cost of U_{ij} is composed of four components; U_{sij}, the savings in system depletions from buying the j-th item in the i-th period; U_{Bij}, the savings in base stockage costs from buying the j-th item in the i-th period; U_{hij}, the holding cost refund associated with buying the j-th item in the i-th period; and U_{oj}, the obsolescence refund associated with buying the j-th item.

We may now define $K_{U_{ij}}$, the cost of activity U_{ij}, as:

12. $K_{U_{ij}} = -(U_{sij} + U_{Bij} + U_{hij} + U_{oj})$

The term $K_{U_{ij}}$ will be either negative or zero.

a. The Derivation of U_{sij}

In order to estimate the system depletion costs, U_{sij}, we shall compute, for any period i of the model, the number of items procured through that period; and the number of items in a reparable or condemned condition in that period. The latter, the number of items reparable or condemned in period i, will be broken down into the number condemned before repair period i, and the number condemned in repair period i and reparable in model period i.

Repair period i will include those periods of the model in which, if a reparable item were to be generated in one of them, it would remain reparable in model period i, given the assumptions of routine and expedited repair.

Thus, if we are using an 8 period model, in which the 8th period is surplus, and are assuming a repair cycle of 3 periods, periods 3, 4, and 5 would be included in repair period 5. Any item demanded in period 2 would be available as a serviceable in period 5. Any item demanded in period 3, however, would not become serviceable until period 6, and would be reparable in period 5.

We shall define our demand probability functions, $f_i(x)$ in terms of the repair period of time. These probability functions, representing exchanges in period i, then will represent the number of items reparable in model period i or condemned during repair period i. We then estimate separately the number of condemnations prior to repair period i. Note that the reparable item generated prior to repair period i becomes a part of both the cumulative demand through period i and the cumulative supply through period i.

We call the definition of demand probability functions in terms of repair periods the "overlapping repair cycle demand." The following table will illustrate the principle of the overlapping repair cycle demand for a 3 period repair cycle, and an 8 period model.

Table 1

Repair Period	Model Periods
1	1
2	1 and 2
3	1, 2, and 3
4	2, 3, and 4
5	3, 4, and 5
6	4, 5, and 6
7	5, 6, and 7
8	Surplus

The overlapping repair cycle demand is the technique used to introduce the availability of serviceable spare items from repair into the model as an endogenous factor.

The impact of expediting repair may be seen quite easily by reference to Table 1. For example, if we were to expedite repair in the 3rd period, assuming a one period expedited repair cycle, the items expedited through repair would become available in the 4th period. Thus, the relevant demand for depletions in the 4th would be demand in the 2nd and 4th, (repair period 4 would consist of model periods 2 and 4), and the relevant demand for depletions in the 5th would be demand in the 4th and 5th (repair period 5 would consist of model periods 4 and 5). For purposes of model 1, that is, for purposes of the routine repair and expedited repair sub-optimizations, we shall assume that the demand in each model period is given as a probability function. We shall now introduce the demand probability function $f_i(x)$ which is the demand probability function formed by combining all of the demand probability functions for the model periods in repair period i. For example, in Table 1, $f_i(x)$ where i is 5, would represent the combined demand probability functions of model periods 3, 4, and 5. With repair expedited in period 3, $f_i(x)$ with $i = 5$, would represent the combined demand probability functions of model periods 4 and 5.

Let us introduce some additional terminology at this time:

(1) U'_{sij} = savings to be obtained <u>in period i</u> from buying the j-th item for delivery in or before period i.

(2) U''_{sij} = savings to be obtained in period i from having the j-th item in the system that is not condemned prior to repair period i.

(3) $f_i(x)$ is the system demand probability function for model periods in repair period i;

(4) $\bar{f}_i(x)$ is the system demand probability function for model periods prior to repair period i;

(5) $f'_i(x)$ is the system demand probability function for all model periods up to and including model period i;

(6) $\bar{w}$ is the probability of an item received in exchange being condemned, = wearout factor or per cent;

(7) t is the number of condemnations, a random variable;

(8) $w_x(t)$ is the condemnation probability function given x issues, =

$$(1-\bar{w})^{x-t}\,\bar{w}^{t}\,\frac{x!}{t!(x-t)!}$$

(9) $C_i(t)$ is the condemnation probability function for model periods prior to repair period $i = \sum_{x=t}^{\infty} w_x(t) \; \bar{f}_i(x)$; [1]

(10) $C_i'(t)$ is the condemnation probability function for all periods in the model up to and including model period i, =

$\sum_{x=t}^{\infty} w_x(t) \; f_i'(x)$. [2]

(11) a_i is the cumulative expected gains of the spare item from attrition and whole aircraft salvage prior to repair period i

(12) a_i' is the cumulative expected gains of the spare item from attrition and whole aircraft salvage up to and including model period i.

Given a repair plan (that is, a plan determining in which periods expedited repair will be used, and in which periods routine repair will be used), and given the demand probability functions for each period in the model, all $f_i(x)$ functions are determinate. For the routine repair and expedited repair sub-optimizations, both conditions are met.

We may now write the following equations:

13. $$U''_{sij} = \sum_{x=j}^{\infty} \pi_{s,x-j+1} \, f_i(x).$$

In this equation, $\pi_{s,x-j+1}$ is the savings from having the j-th item in the system if there are x issues in the repair period; since if that item were not in the system, there would have been $x-j+1$ depletions, and with that item in the system, there are only $x-j$ depletions.

---

[1] If $\bar{f}_i(x)$ is poisson, with mean m_i, then $C_i(t)$ is poisson with mean $\bar{w}m_i$.

[2] If $f_i'(x)$ is poisson, with mean m_i', then $C_i'(t)$ is poisson with mean $\bar{w}m_i'$.

The term $f_i(x)$ is the probability of issuing x in the repair cycle. The summation sums the product of the probability of each x and the savings, given each x, from $x = j$, at which point the first savings occur, to $x = \infty$.

$$14. \quad U'_{sij} = \sum_{t=0}^{\infty} U''_{si,\,j+a_i-t}\, C_i(t)\,.$$

Equation (14) translates the savings of the various j-th items <u>in the system</u> in period i (the U''_{sij}'s) into the savings of the j-th item <u>purchased</u> for delivery through period i. In this equation, $j+a_i-t$ is the number of item left in the system (1st, 2nd, 3rd, etc.) that the j-th item purchased will be if there are t condemnations prior to repair period i; $U''_{si,j+a_i-t}$ is the saving which will result from the j-th item bought if there are t condemnations; and $C_i(t)$ is the probability of t condemnations. The summation sums the product of savings, given t condemnations, and probability of t condemnations, from $t = 0$ to ∞.

$$15. \quad U_{sij} = \sum_{k=i}^{n} U'_{skj}$$

Equation (15) merely sums the period savings from buying the j-th item for delivery in the i-th period, for all periods from i to n.

Substituting equation (13) in equation (14), and then equation (14) in equation (15), we may define U_{sij} as:

$$16. \quad U_{sij} = \sum_{k=i}^{n} \sum_{t=0}^{\infty} \sum_{x=j-t+a_k}^{\infty} \pi_{s,x-j+t-a_k+1}\, f_k(x)\, C_k(t),$$

b. The Derivation of U_{hij}

Let us introduce some new terminology at this time.

1. ${}_a h_i$ is the cost in the i-th time period caused by holding an item that is in the system as a serviceable or reparable;

$= h_i\ (1-\Omega_1)\ (1-\Omega_2)\ (1-\Omega_3)\ ------\ (1-\Omega_i)$ where

Ω_k is the probability of engineering change obsolescence in period k.

We may now write an equation for U_{hij} as:

$$17.\quad U_{hij} = P_{hi} - \sum_{k=i}^{n} {}_a h_k \sum_{t=0}^{j+a'_k-1} C'_k(t).$$

In this equation, P_{hi} is the holding cost charged as part of the P cost; $\sum_{k=i}^{n} {}_a h_k \sum_{t=0}^{j+a'_k-1} C'_k(t)$ is the ~~true~~ expected holding cost; and the difference represents the refund to be inserted as part of the U cost.

In $\sum_{k=i}^{n} {}_a h_k \sum_{t=0}^{j+a'_k-1} C'_k(t)$, the term $\sum_{t=0}^{j+a'_k-1} C'_k(t)$ is the cumulative probability of condemning less than $j+a'_k$, in which case the holding cost in period k, ${}_a h_k$, will be incurred. The summation $\sum_{k=1}^{n}$ sums these cost-probability products for all periods from i to n.

c. The Derivation of U_{oj}, The Obsolescence Refund

If the Air Force is willing to phase its aircraft out as incomplete aircraft, U_{oj} must be set at zero. If the Air Force is unwilling to phase its aircraft out as whole aircraft, equation (19) should be used to value U_{oj}. We have charged the procurement unit cost against all items procured in the development of the P costs. The obsolescence refund is a device for compensating for the overcharge in the P costs.

The use of a full charge of unit cost as part of the P cost and a refund for the probability of condemnation as part of the U cost is equivalent to charging the unit cost times the probability of non-condemnation (or, in other words, the probability of obsolescence). In fact, $b^{P}j$ is defined in just that way in equation (18).

Let us introduce some new terminology at this time:

1. $b^{P}j$ is the true cost of procurement of the j-th item.

We may now write the following equations:

18. $$b^{P}j = P_{p,n-1} \sum_{t=0}^{j+a_n'-1} C_n'(t).$$

In this equation, the term $\sum_{t=0}^{j+a_n'-1} C_n'(t)$ represents the probability of condemning less than $j+a_n'$ in the lifetime of the spare item. It is thus the probability that the j-th item bought will incur terminal obsolescence.

19. $$U_{oj} = P_{p,n-1} - b^{P}j = P_{p,n-1} \sum_{t=j+a_n'}^{\infty} C_n'(t).$$

In this equation, $P_{p,n-1} - b^{P}j$ represents the difference between the procurement unit cost charged as part of the P cost (in the last period) and the true procurement unit cost, and thus is the refund to be credited as part of the U cost. The term $P_{p,n-1} \sum_{t=j+a_n'}^{\infty} C_n'(t)$ represents the unit cost times the probability of condemnation, and is mathematically equivalent to $P_{p,n-1} - b^{P}j$.

d. The Derivation of U_{Bij}

Let us introduce some new terminology at this time:

1. $g_i(y)$ = the savings in base stockage in period i from having the y-th item in a serviceable condition in period $i - P_d$ where P_d is the depot-to-base routine pipeline.

2. $H_{ij}(y)$ = the probability function of having y items in a serviceable condition in period $i - P_d$ if j items are delivered from procurement through period $i - P_d$.

3. U'_{Bij} is the savings in period i in base stockage costs from buying the j-th item for delivery in or before the $i - P_d$th period.

4. U''_{Bij} is the total savings in period i in base stockage costs from buying j items (items 1 through j) for delivery in or before the $i - P_d$th period.

5. $f_i^k(x)$ = the demand probability function in period i at base k = the probability of demanding x in period i at base k.

The $g_i(y)$ function may be obtained through the use of any base optimal stockage solution that is desired.[1] For these models, however, we shall use a simplified approximation based on the base depletion penalty and base depletion probabilities only. This simplified approximation is sufficiently accurate for high-cost low-demand items. It operates on the assumption that all serviceable items will be stocked at base level.

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1 We recommend, for example, Optimal Inventory Policy for a Military Organization, E. B. Berman and A. J. Clark, P-647, March, 1955.

The terms $\sum_{x=y}^{\infty} \pi_{B,x-y+1} f_i^k(x)$, which are the savings from putting the y-th serviceable item at base k in period i, are computed for all bases and all quantities y.

These terms are then rearranged, within each period i, in descending order of magnitude. The reordered terms form the $g_i(y)$ function.[1]

The $H_{ij}(y)$ function may be defined as follows:

$$20. \quad H_{ij}(y) = \sum_{t=0}^{j-y+a_{i-P_d}} C_{i-P_d}(t)\, f_{i-P_d}(j - t + a_{i-P_d} - y)$$

In this equation, $f_{i-P_d}(j - t + a_{i-P_d} - y)$ is the probability of a demand of $j - t + a_{i-P_d} - y$ in repair period $i-P_d$. This would leave y items in a serviceable condition, if t were condemned prior to repair period $i - P_d$. $C_{i-P_d}(t)$ is the probability of t being condemned prior to repair period $i - P_d$. The summation sums the product of the two probabilities over all t from 0 to $j - y + a_{i-P_d}$ at which level, $f_{i-P_d}(x)$ is forced to zero.

We may now define U''_{Bij} as:

$$21. \quad U''_{Bij} = \sum_{y=1}^{j+a_{i-P_d}} H_{ij}(y) \sum_{z=1}^{y} g_i(z)$$

In this equation, $\sum_{z=1}^{y} g_i(z)$ represents the total savings in base

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[1] If the identification of the base is retained in the $g_i(y)$ function, this function may later be used to distribute the items which are actually serviceable in period $i - P_d$ among the bases.

costs of having y items at the base, and $H_{ij}(y)$ is the probability of having y items at the base. The first summation, $\sum_{y=1}^{j+a_{i-P_d}}$, sums the cost-probability products for all y from 1 to $j+a_{i-P_d}$, which is the maximum possible amount of serviceables in the system in period $i-P_d$.

U'_{Bij} may now be defined as:

22. $U'_{Bij} = U''_{Bij} - U''_{Bi,j-1}$

This equation states merely that the savings in the i-th period from receiving delivery of the j-th item in or before the $i-P_d$th period is the total savings of having received j items in delivery by $i-P_d$ minus the total savings of having received only $j-1$ items.

U_{Bij} may now be written as:

23. $U_{Bij} = \sum_{k=i+P_d}^{n} U'_{Bkj}$

This equation merely sums the savings in base costs from buying the j-th item for delivery in the i-th period over all periods from $i+P_d$ to n.

Substituting equation (20) into equation (21); equation (21) into equation (22); and equation (22) into equation (23), we may define U_{Bij} as:

24. $U_{Bij} = \sum_{k=i+P_d}^{n} \left[\sum_{y=1}^{j+a_{k-P_d}} \quad \sum_{z=1}^{y} g_k(z) \sum_{t=0}^{j-y+a_{k-P_d}} C_{k-P_d}(t)\, f_{k-P_d}(j-t+a_{k-P_d}-y) \right.$

$$- \sum_{y=1}^{j+a_{k-P_d}-1} \sum_{z=1}^{y} g_k(z) \sum_{t=0}^{j-y+a_{k-P_d}-1} C_{k-P_d}(t) \; f_{k-P_d}(j-t+a_{k-P_d}-y-1) \Big]$$

Costs for Model 2

As noted in Chapter I, the costs for R_{ko} and S_k are zero, since they are surplus activities.

The costs of activities R_k, and $R_{k,k-1}$, which we shall call respectively, K_{R_k} and $K_{R_{k,k-1}}$, will be developed in two portions: R'_k and $R'_{k,k-1}$, which represent the additional costs incurred by expediting repair, including therein, additional costs of packing, transporting, inspecting, handling, and overhauling; and V_k and $V_{k,k-1}$ which represent the savings (or negative costs) of expediting repair, including therein, the reduced probability of system depletion and the reduced costs of stocking at the base.

We may write the equations:

25. $K_{R_k} = R'_k + V_k$

26. $K_{R_{k,k-1}} = R'_{k,k-1} + V_{k,k-1}$

1. The Development of R'_k and $R'_{k,k-1}$

The costs R'_k and $R'_{k,k-1}$ are always equal, since we may assume that the cost of expediting repair on reparables generated in period k is independent of whether or not repair was expedited on reparables generated in the preceding period.

Let us introduce some new terminology at this time:

1. R_k^r is the cost per item of performing routine repair on all reparables generated in period k. The value of R_k^r will be zero if k is within the routine repair cycle pipeline of the last active period of the model. In other words, if there are 8 periods in the model, of which the first 7 are active and the last is a surplus period, and if the routine repair cycle is 3 periods, routine repair would not be performed in periods 5, 6, 7, and 8; since no item on which repair was initiated in those periods could be available for an active period of the model. Therefore, the value of R_k^r when k equals 5 through 8 would be zero.

2. R_k^e is the total cost per item of performing expedited repair on all reparables generated in period k.

3. $\bar{x}_k$ is the expected demand in model period (not repair period) k.

4. $\bar{a}_k$ is the expected gain of spare items in model period k from attrited and salvaged aircraft.

We may now write the equation for R'_k and $R'_{k,k-1}$:

27. $R'_k = R'_{k,k-1} = \left[\bar{x}_k \, (1 - \bar{w}) + \bar{a}_k \right] (R_k^e - R_k^r)$, where the bracketed term is the expected generation of reparables in period k, and $(R_k^e - R_k^r)$ is the net unit cost of expediting repair.

2. The Development of V_k and $V_{k,k-1}$

The V costs will be developed on the assumptions stated for model 2, that the routine repair cycle is 3 periods, and that the expedited repair cycle is one period.

The V costs are savings resulting from expediting repair, while holding procurement fixed. Therefore, these savings are limited to

U_{sij}, the system depletion costs, and U_{Bij}, the base stockage costs; the holding, procurement, and obsolescence costs are not involved.

In developing V costs, we shall assume that if there can be savings in period i from expediting repair in both period $i-1$ and period $i-2$, we shall assess the savings from the earlier period $(i-2)$ first, and, in case repair is expedited in both $i-1$ and $i-2$, shall treat the savings from expediting repair in $i-1$ as the marginal or additional savings.

Let us introduce some new terminology at this time:

1. ${}_{k+1}V^{m}_{sk}$ is the saving in system depletions to be obtained in period $k+1$ by expediting repair on reparables generated in period k when repair is not expedited on reparables generated in period $k-1$, and if m items are delivered from procurement through period $k+1$;

2. ${}_{k+2}V^{m}_{sk}$ is the savings in system depletions to be obtained in period $k+2$, if m items are delivered from procurement through period $k+2$;

3. ${}_{k+1}V^{m}_{sk,k-1}$ is the savings in system depletions to be obtained in period $k+1$ by expediting repair on reparables generated in period k, if repair is also expedited on reparables generated in period $k-1$, and if m items are delivered from procurement through period $k+1$;

4. ${}_{k+P_d+1}V^{m}_{Bk}$ is the savings in base stockage costs in period $k+P_d+1$ from expediting repair in period k, if repair is not expedited in period $k-1$, and if m items are delivered from procurement through period $k+1$;

5. ${}_{k+P_d+2}V^{m}_{Bk}$ is the savings in base stockage costs to be obtained in period $k+P_d+2$ by expediting repair on reparables generated in period k, if m items are delivered from procurement through period $k+2$;

6. ${}_{k+P_d+1}V^m_{Bk,k-1}$ is the savings in base stockage costs to be obtained in period $k+P_d+1$ by expediting repair in period k if repair has been expedited on reparables generated in period $k-1$, and if m items are delivered from procurement through period $k+1$;

7. A presuperscript 1 before U'_{sij} will identify this cost as having been developed under the assumption of no expediting of repair in both period $i-2$ and period $i-1$; this presuperscript before U''_{Bij} will identify this cost as having been developed under the assumption of no expediting of repair in both period $i-2-P_d$ and period $i-1-P_d$.

8. A presuperscript 2 before U'_{sij} will identify this cost as been developed under the assumption of repair being expedited in period $i-2$, but not in period $i-1$; this presuperscript before U''_{Bij} will identify this cost as having been developed under the assumption of repair being expedited in period $i-2-P_d$, but not in period $i-1-P_d$.

9. A presuperscript 3 before U'_{sij} will identify this cost as having been developed under the assumption of repair being expedited in period $i-1$ but not in period $i-2$; this presuperscript before U''_{Bij} will identify this cost as having been developed under the assumption of repair being expedited in period $i-1-P_d$, but not in period $i-2-P_d$.

10. A presuperscript 4 before U'_{sij} will identify this cost as having been developed under the assumption of repair being expedited in both period $i-2$ and period $i-1$; this presuperscript before U''_{Bij} will identify this cost as having been developed under the assumption of repair being expedited in both period $i-2-P_d$ and period $i-1-P_d$.

We may now write the following equations:

28. $_{k+1}V^{m}_{sk} = \sum_{p=m+1}^{q} {}^{1}U'_{s,k+1,p} - \sum_{p=m+1}^{q} {}^{3}U'_{s,k+1,p}$

where q is the number of U columns and m is procurement through the presubscript period.

29. $_{k+2}V^{m}_{sk} = \sum_{p=m+1}^{q} {}^{1}U'_{s,k+2,p} - \sum_{p=m+1}^{q} {}^{2}U'_{s,k+2,p}$

30. $_{k+1}V^{m}_{s,k,k-1} = \sum_{p=m+1}^{q} {}^{2}U'_{s,k+1,p} - \sum_{p=m+1}^{q} {}^{4}U'_{s,k+1,p}$

31. $_{k+P_d+1}V^{m}_{Bk} = {}^{3}U''_{B,k+P_d+1,m} - {}^{1}U''_{B,k+P_d+1,m}$

where m is procurement through the presubscript period minus P_d.

32. $_{k+P_d+2}V^{m}_{Bk} = {}^{2}U''_{B,k+P_d+2,m} - {}^{1}U''_{B,k+P_d+2,m}$

33. $_{k+P_d+1}V^{m}_{B,k,k-1} = {}^{4}U''_{B,k+P_d+1,m} - {}^{2}U''_{B,k+P_d+1,m}$

34. $V_k = -\left({}_{k+1}V^{m}_{sk} + {}_{k+2}V^{m}_{sk} + {}_{k+P_d+1}V^{m}_{Bk} + {}_{k+P_d+2}V^{m}_{Bk}\right)$

35. $V_{k,k-1} = -\left({}_{k+1}V^{m}_{s,k,k-1} + {}_{k+2}V^{m}_{sk} + {}_{k+P_d+1}V^{m}_{B,k,k-1} + {}_{k+P_d+2}V^{m}_{Bk}\right)$

Note in equations (34) and (35) that V_k and $V_{k,k-1}$ the alternative savings from expediting repair in period k, each consists of four parts. In each case, the first two parts are savings from reductions in the probability of system depletions in periods k+1 and k+2. These system savings are the only ones which could possibly result from expediting repair on reparables generated in period k. In period k, reparables generated in period k will still be reparable whether they are undergoing routine or expedited repair. In period k+3, reparables generated in period k will be serviceable whether they were repaired using the expedited or the routine repair cycle.

The last two parts of V_k and $V_{k,k-1}$ represent the savings in base stockage costs. Here too, only two periods are relevant, although the periods are time-phased ahead by an amount P_d, the routine depot-base pipeline.

Equations (28) through (33) define the various parts of V_k and $V_{k,k-1}$. In equations (28) through (30), each system saving is defined as the expected uncovered depletion cost in that period if repair is not expedited in period k, minus the expected uncovered depletion cost in that period if repair is expedited in period k. In equations (31) through (33), each base saving is defined as the total base stockage saving in the period if repair is expedited in period k, minus the total base stockage saving in the period if repair is not expedited in period k.

Comparing Total Costs of Sub-Optimizations

In costing solutions of sub-optimizations for comparison purposes, we must keep in mind the fact that the U costs are not comparable in the two sub-optimizations unless the repair plans (i.e., the plans for expediting repair by period) are identical. Therefore, we must use the following procedure for computing the cost of models:

a. Cost Computation for Solution of a Routine Repair or Expedited Repair Sub-Optimization

Let us define one additional terminology:

1. σ_i = the cumulative deliveries from procurement through period i.

We may write the equation for T, the total cost of a routine repair or expedited repair sub-optimization, as:

$$36. \quad T = \sum_{i=1}^{n} P_i K_{P_i} + \sum_{i=1}^{n} \sum_{j=\sigma_i+1}^{q} U'_{sij}$$

$$+ \sum_{i=1}^{n} \sum_{j=\sigma_{i-P_d}+1}^{q} U'_{Bij} - \sum_{i=1}^{n} \sum_{j=1}^{q} U_{ij} (U_{hij} + U_{oj})$$

$$+ \sum_{k=1}^{n-2} R_k R'_k + \sum_{k=2}^{n-2} R_{k,k-1} R'_{k,k-1};$$

where n is the number of periods in the model;

q is the number of U columns;

and the R activity levels are 1 for the expedited repair sub-optimization and zero for the routine repair sub-optimization.

In this equation, the first term in the right-hand side of the equation is the procurement cost, where the K_{P_i} are the unit costs of procurement and the P_i are the activity levels; the fourth contains both the obsolescence and holding cost refunds, where $(U_{hij} + U_{oj})$ represents the refunds and the U_{ij} are the activity levels; and the fifth and sixth contain the net costs of expediting repair, where the R'_k and $R'_{k,k-1}$ are the costs and the R_k and $R_{k,k-1}$ are the activity levels.

The second term costs the uncovered system depletions, and the third term costs the uncovered base depletions. This procedure is used since uncovered depletion costs are comparable between sub-optimizations.

III

THE VALUE OF DEFERRING DECISIONS

We shall suggest in this chapter a technique for estimating the value of deferring decisions, and for using this value in the model once it is estimated. The technique requires the use of a Monte Carlo model; we shall not, however, describe the Monte Carlo model which should be used.[1]

In order to estimate the value of deferring decisions, we should perform the routine repair and expedited repair sub-optimization using model 3, which is shown in Figure 3.

Figure 3

MODEL 3

Column Totals	A_1	A_2	A_3	A_4	A_5	A_6	1	1	1	1	1	
i	P_{1j}	P_{2j}	P_{3j}	P_{4j}	P_{5j}	P_{6j}	U_{1j}	U_{2j}	U_{3j}	U_{4j}	U_{5j}	Row Totals
1	P_{11}	x	x	x	x	x	U_{11}	U_{12}	U_{13}	U_{14}	U_{15}	0
2	P_{21}	P_{22}	x	x	x	x	U_{21}	U_{22}	U_{23}	U_{24}	U_{25}	0
3	P_{31}	P_{32}	P_{33}	x	x	x	U_{31}	U_{32}	U_{33}	U_{34}	U_{35}	0
4	P_{41}	P_{42}	P_{43}	P_{44}	x	x	U_{41}	U_{42}	U_{43}	U_{44}	U_{45}	0
5	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	x	U_{51}	U_{52}	U_{53}	U_{54}	U_{55}	0
6(s)	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	U_{61}	U_{62}	U_{63}	U_{64}	U_{65}	

Criterion: $\sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} K_{P_{ij}} + \sum_{i=1}^{n} \sum_{j=1}^{p} U_{ij} K_{U_{ij}} = \text{MIN.}$

Where $K_{P_{ij}}$ is the cost of activity P_{ij}

$K_{U_{ij}}$ is the cost of activity U_{ij}

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[1] A Monte Carlo model which could be used with this technique is now being developed by the Systems Group, Logistics Section, of The RAND Corporation.

n is the number of periods in the study

m is the number of P columns

and p is the number of U columns

In Model 3, the U side of the double line is identical to the U side of Model 1.

P_{ij} is the amount of items which are ordered in period j and delivered in period i. The x cells indicate that it is impossible to receive delivery in the row period if the order is placed in the column period.

The ordering periods need not be identical to the delivery period. In fact, it is intended that the first ordering period be the one in which an order must be placed if delivery, assuming expedited procurement, is to occur in delivery period 1.

We may write the following equations for Model 3:

37. $\sum_{i=1}^{n} P_{ij} = A_j$; j goes from 1 to m (m equations)

where n is the number of periods in the model, including surplus; and m is the number of P columns

38. $\sum_{i=1}^{n} U_{ij} = 1$; j goes from 1 to p (p equations)

where p is the number of U columns

39. $\sum_{j=1}^{m} P_{ij} - \sum_{j=1}^{p} U_{ij} = 0$; i goes from 1 to $n-1$ ($n-1$ equations).

The A_j are as before the limits on the amount of procurement, except that in Model 3, these limits are imposed on the amount of items ordered in any one period. This breakdown of the procurement limit makes Model 3 useful for determining the procurement-repair decision under a maximum-size-of-order-per-period constraint.

The costs of activities in Model 3 are very similar to those in Model 1. The U costs are identical. The costs of the P_1 column are identical to the costs of the P column in model 1. In the other P columns, the expedited procurement penalty will be added to the P cost in one additional cell for each column moved to the right from P_1. As a general rule, we may write the following equation:

40. $P_{eij} = P_{e,i+1,j+1}$

where P_{eij} is as before the expedited procurement penalty.

If the only reason for using Model 3 was to solve for the procurement-repair decision under a maximum-size-of-order constraint, the adjustment in the P costs given by equation (40) would be sufficient.[1] If, however, we wish to use Model 3 to test for the value of decision deferment, we must make a further adjustment in the P_{ij} cost. We insert a new element into the model, which we may call d_j; this element is the value of decision deferment. We add d_j to each P_{ij} cost. It is suggested that d_j be set equal to $d' \sum_{q=j}^{m} \frac{E_q - E_{q-1}}{E_q}$ where d' is a coefficient which we shall vary in the Monte Carlo model, m is the number of ordering periods, and E_j is the experience in programmed flying hours of the applicable air-aircraft, up to and including ordering period j. This particular formulation of d_j results in a value of decision deferment which will vary

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[1] Model 3 is, like Model 1, transportation in form, and solves as easily.

with the percentage increase in experience $\left(\frac{E_q - E_{q-1}}{E_q}\right)$. Then we may run the model (that is, Model 3 with appropriate reverse sub-optimizations) against a Monte Carlo model, using various sizes of d', to determine which size of d' gives the preferred result.

Actually, d_j should be inserted in all applications of Model 3. A d_j approaching zero will serve to defer orders automatically as long as the deferment is free.

IV

AN EXAMPLE

We shall perform a routine repair sub-optimization; an expedited repair sub-optimization; and a reverse sub-optimization on the lower cost of the routine and expedited repair sub-optimizations; for a hypothetical item.

1. Assumptions of Example

1. We shall assume that 5 periods of activity represent the lifetime use of the spare item. There will be a 6th period (n=6) for surplus.
2. π_{sq}, the system depletion penalty = \$1,000 per period for $q \leq 2$; = \$5,000 for $q > 2$.
3. π_{bq}, the base depletion penalty = \$200
4. The routine repair cycle is 3 periods.
5. The expedited repair cycle is 1 period.
6. $\overline{w}$, the wearout factor or per cent = 20 per cent per exchange.
7. The unit cost = \$50.
8. The expedited procurement penalty = \$50.
9. The holding cost, h_k, = \$10 per period.
10. The emergency depot-to-base pipeline = 0 periods.
11. P_d, the routine depot-to-base pipeline = 1 period.
12. Delivery in the 1st period requires expedited procurement; in all other periods, routine procurement may be used.
13. There are 2 bases; base 1 is active in all periods; base 2 is active in periods 3 and 4.
14. The probability function at each base in an active period is given as:

x	$f_i^k(x)$
0	.5
1	.5

15. The system demand is the sum of base demands in each period; there is no additional demand.

16. a_i, the expected gains from attrition and salvage, = 0 for all i.

17. Expedited repair costs $40 per item.

18. Routine repair costs $10 per item.

19. The probability of engineering change obsolescence in period i, Ω_i, = 0 for all i.

20. The applicable aircraft need not be phased out as complete aircraft.

2. Some Basic Tables

In this section, we shall use the presuperscript 1 to mean that repair is routine in model periods i-2 and i-1; the presuperscript 2 to mean that repair is expedited in model period i-2, but not in model period i-1 ; the presuperscript 3 to mean that repair is expedited in model period i-1, but not in model period i-2; and the presuperscript 4 to mean that repair is expedited in both model periods i-2 and i-1.

Table 2[1]

$w_x(t)$

t \ x	0	1	2	3	4	5	6	7
0	1.0	.8	.64	.512	.4096	.32768	.262144	.2097152
1	0	.2	.32	.384	.4096	.4096	.393216	.3670016
2	0	0	.04	.096	.1536	.2048	.24576	.2752512
3	0	0	0	.008	.0256	.0512	.08192	.1146880
4	0	0	0	0	.0016	.0064	.015360	.0286720
5	0	0	0	0	0	.00032	.001536	.0043008
6	0	0	0	0	0	0	.000064	.0003584
7	0	0	0	0	0	0	0	.0000128

1 Derived from assumption 6; see P. 14a.

Table 3[1]

$C_i'(t)$

* = less than .00005

Rounded to four decimals

i \ t	0	1	2	3	4	5	6	7
1	.9	.1	0	0	0	0	0	0
2	.81	.18	.01	0	0	0	0	0
3	.6561	.2916	.0486	.0036	.0001	0	0	0
4	.5314	.3543	.0984	.0146	.0012	*	*	0
5	.4783	.3720	.1240	.0230	.0026	.0002	*	*

[1] Tables 3 through 7 were derived from assumptions 4, 5, 13, 14, and 15; and Table 2. See p. 15.

Table 4

${}^1C_i(t)$

i \ t	0	1	2
1	1.0	0	0
2	1.0	0	0
3	1.0	0	0
4	.9	.1	0
5	.81	.18	.01

Table 5

$^{2}C_i\ (t)$

i \ t	0	1	2	3	4
1	1.0	0	0	0	0
2	1.0	0	0	0	0
3	.9	.1	0	0	0
4	.81	.18	.01	0	0
5	.6561	.2916	.0486	.0036	.0001

Table 6

$^{3}C_i\ (t)$

i \ t	0	1	2	3	4
1	1.0	0	0	0	0
2	.9	.1	0	0	0
3	.9	.1	0	0	0
4	.729	.243	.027	.001	0
5	.6561	.2916	.0486	.0036	.0001

Table 7

$^{4}c_i\ (t)$

Rounded to four decimals

i \ t	0	1	2	3	4	5	6
1	1.0	0	0	0	0	0	0
2	.9	.1	0	0	0	0	0
3	.81	.18	.01	0	0	0	0
4	.6561	.2916	.0486	.0036	.0001	0	0
5	.5314	.3543	.0984	.0146	.0012	*	*

* = less than .00005

Table 8 [1/]

$^{1}f_i\ (x)$

i \ x	1	2	3	4	5	Model Periods in Repair Period i
1	.50	0	0	0	0	1
2	.50	.25	0	0	0	1,2
3	.2500	.3750	.2500	.0625	0	1,2,3
4	.15625	.31250	.31250	.15625	.03125	2,3,4
5	.15625	.31250	.31250	.15625	.03125	3,4,5
6	0	0	0	0	0	——

[1/] Tables 8,9,10, and 11 were derived from assumptions 4,5,13,14, and 15; see pp. 12 ff.

Table 9

$^2f_i\ (x)$

i \ x	1	2	3	4	5	Model Periods in Repair Period i
1	.50	0	0	0	0	1
2	.50	.25	0	0	0	1,2
3	.375	.375	.125	0	0	2,3
4	.2500	.3750	.2500	.0625	0	3,4
5	.375	.375	.125	0	0	4,5
6	0	0	0	0	0	—

Table 10

$^3f_i\ (x)$

i \ x	1	2	3	4	5	Model Periods in Repair Period i
1	.50	0	0	0	0	1
2	.50	0	0	0	0	2
3	.375	.375	.125	0	0	1,3
4	.375	.375	.125	0	0	2,4
5	.375	.375	.125	0	0	3,5
6	0	0	0	0	0	—

Table 11

$^4f_i\ (x)$

i \ x	1	2	3	4	5	Model Periods in Repair Period i
1	.50	0	0	0	0	1
2	.50	0	0	0	0	2
3	.50	.25	0	0	0	3
4	.50	.25	0	0	0	4
5	.50	0	0	0	0	5
6	0	0	0	0	0	—

Table 12 1/

$g_i\ (y)$ in Dollars

i \ y	1	2
1	100	0
2	100	0
3	100	100
4	100	100
5	100	0
6	0	0

1/ See The Derivation of U_{Bij}, pp. 19 ff

Table 13 1/

$\sum_{z=1}^{y} g_i(z)$ in Dollars

$i \backslash y$	1	2	3	4	5	6	7
1	100	100	100	100	100	100	100
2	100	100	100	100	100	100	100
3	100	200	200	200	200	200	200
4	100	200	200	200	200	200	200
5	100	100	100	100	100	100	100
6	0	0	0	0	0	0	0

1/ Derived from Table 12.

3. Costs Which Apply to All Sub-Optimizations

In this section we shall derive K_{pi}, U_{hij} and $U_{o\,j}$

Table 14 1/

K_{pi} in Dollars

i	P_{hi}	P_{pi}	P_{ei}	K_{pi}
1	50	50	50	150
2	40	50	0	90
3	30	50	0	80
4	20	50	0	70
5	10	50	0	60
6	0	0	0	0

1/ Derived from assumptions 7, 8, 9, and 12, using equations (12) and (13).

Table 15 [1/]

$$\sum_{t=0}^{j+a_k'-1} C_k'(t)$$

Rounded to four decimals.

k \ j	1	2	3	4	5	6	7
1	.9	1.0	1.0	1.0	1.0	1.0	1.0
2	.81	.99	1.0	1.0	1.0	1.0	1.0
3	.6561	.9477	.9963	.9999	1.0000	1.0000	1.0000
4	.5314	.8857	.9841	.9987	.9999	1.0000	1.0000
5	.4783	.8503	.9743	.9973	.9999	1.0000	1.0000

1/ Derived from assumption 16, and Table 3; see p. 17.

Table 16 [1/]

$$\sum_{k=i}^{n} a^h{}_k \sum_{t=0}^{j+a'_k-1} C_k'(t) \text{ in Dollars}$$

Rounded to the nearest cent

i \ j	1	2	3	4	5	6	7
1	33.76	46.74	49.55	49.96	50.00	50.00	50.00
2	24.76	36.74	39.55	39.96	40.00	40.00	40.00
3	16.66	26.84	29.55	29.96	30.00	30.00	30.00
4	10.10	17.36	19.58	19.96	20.00	20.00	20.00
5	4.78	8.50	9.74	9.97	10.00	10.00	10.00

1/ Derived from assumptions 9 and 19, and Table 16; see p. 17.

Table 17 1/

U_{hij} in Dollars Rounded to the nearest cent

i \ j	1	2	3	4	5	6	7
1	16.24	3.26	0.45	0.04	0	0	0
2	15.24	3.26	0.45	0.04	0	0	0
3	13.34	3.16	0.45	0.04	0	0	0
4	9.90	2.64	0.42	0.04	0	0	0
5	5.22	1.50	0.26	0.03	0	0	0

1/ Derived from Tables 14 and 16, using equation 17.

$U_{oj} = 0$ for all j; see assumption 20 and p. 17.

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4. Costs for Routine Repair Sub-Optimization

In this section, we shall develop K_{Uij} for the routine repair sub-optimization.

Table 18 1/

${}^{1}U''_{sij}$ in Dollars

i \ j	-1	0	1	2	3	4	5
1	3,000.00	1,000.00	500.00	0	0	0	0
2	4,000.00	2,000.00	750.000	250.00	0	0	0
3	4,750.00	3,750.00	2,187.50	.937.50	312.50	62.50	0
4	4,875.00	4,250.00	2,968.75	1,562.50	625.00	187.50	31.25
5	4,875.00	4,250.00	2,968.75	1,562.50	625.00	187.50	31.25

1/ Table 18 was developed from assumption 2 and Table 8, using equation 13.

Table 19 1/

$^{1}U'_{sij}$ in Dollars

Rounded to nearest cent.

* = less than 0.005

i \ j	1	2	3	4	5	6	7
1	500	0	0	0	0	0	0
2	750	250	0	0	0	0	0
3	2,187.50	937.50	312.50	62.50	0	0	0
4	3,096.88	1,703.13	718.75	231.25	46.88	3.13	0
5	3,218.44	1,842.50	817.19	280.00	65.31	7.50	*

1/ This Table was developed from Tables 4 and 18 and assumption 16, using equation 14.

Table 20 1/

$^{1}U_{sij}$ in Dollars

Rounded to nearest cent.

* = less than 0.005

i \ j	1	2	3	4	5	6	7
1	9,752.82	4,733.13	1,848.44	573.75	112.19	10.63	*
2	9,252.82	4,733.13	1,848.44	573.75	112.19	10.63	*
3	8,502.82	4,483.13	1,848.44	573.75	112.19	10.63	*
4	6,315.32	3,545.63	1,535.94	511.25	112.19	10.63	*
5	3,218.44	1,842.50	817.19	280.00	65.31	7.50	*

1/ This Table was developed from Table 19, using equation 15.

Table 21 [1]

$^{1}H_{ij}$ (y) where i=2

j \ y	1	2	3	4	5	6	7	8
1	.5	0	0	0	0	0	0	0
2	.5	.5	0	0	0	0	0	0
3	0	.5	.5	0	0	0	0	0
4	0	0	.5	.5	0	0	0	0
5	0	0	0	.5	.5	0	0	0
6	0	0	0	0	.5	.5	0	0
7	0	0	0	0	0	.5	.5	0
8	0	0	0	0	0	0	.5	.5

1/ Tables 20-23 were developed from Tables 4 and 8; and assumptions 11 and 16; using equation (20).

Table 22

$^{1}H_{ij}$ (y) where i=3

j \ y	1	2	3	4	5	6	7	8
1	.25	0	0	0	0	0	0	0
2	.50	.25	0	0	0	0	0	0
3	.25	.50	.25	0	0	0	0	0
4	0	.25	.50	.25	0	0	0	0
5	0	0	.25	.50	.25	0	0	0
6	0	0	0	.25	.50	.25	0	0
7	0	0	0	0	.25	.50	.25	0
8	0	0	0	0	0	.25	.50	.25

Table 23

$^{1}H_{ij}(y)$ where i=4

j \ y	1	2	3	4	5	6	7	8
1	.0625	0	0	0	0	0	0	0
2	.2500	.0625	0	0	0	0	0	0
3	.3750	.2500	.0625	0	0	0	0	0
4	.2500	.3750	.2500	.0625	0	0	0	0
5	.0625	.2500	.3750	.2500	.0625	0	0	0
6	0	.0625	.2500	.3750	.2500	.0625	0	0
7	0	0	.0625	.2500	.3750	.2500	.0625	0
8	0	0	0	.0625	.2500	.3750	.2500	.0625

Table 24

$^{1}H_{ij}(y)$ where i=5

j \ y	1	2	3	4	5	6	7	8
1	.028125	0	0	0	0	0	0	0
2	.143750	.028125	0	0	0	0	0	0
3	.296875	.143750	.028125	0	0	0	0	0
4	.312500	.296875	.143750	.028125	0	0	0	0
5	.171875	.312500	.296875	.143750	.028125	0	0	0
6	.043750	.171875	.312500	.296875	.143750	.028125	0	0
7	.003125	.043750	.171875	.312500	.296875	.143750	.028125	0
8	0	.003125	.043750	.171875	.312500	.296875	.143750	.028125

Table 25 1/

$^{1}U''_{Bij}$ in Dollars

Rounded to the nearest cent.

i \ j	1	2	3	4	5	6	7	8
2	50.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
3	25.00	100.00	175.00	200.00	200.00	200.00	200.00	200.00
4	6.25	37.50	100.00	162.50	193.75	200.00	200.00	200.00
5	2.81	17.19	46.88	78.13	95.31	99.69	100.00	100.00
6	0	0	0	0	0	0	0	0

1/ This Table was developed from assumption 16 and Tables 13, 21, 22, 23, and 24, using equation (21).

Table 26 1/

$^{1}U'_{Bij}$ in Dollars

i \ j	1	2	3	4	5	6	7
2	50.00	50.00	0	0	0	0	0
3	25.00	75.00	75.00	25.00	0	0	0
4	6.25	31.25	62.50	62.50	31.25	6.25	0
5	2.81	14.38	29.69	31.25	17.18	4.38	0.31
6	0	0	0	0	0	0	0

1/ This table was developed from Table 25, using equation (22).

Table 27 1/

$^{1}U_{Bij}$ in Dollars

i \ j	1	2	3	4	5	6	7
1	84.06	170.63	167.19	118.75	48.43	10.63	0.31
2	34.06	120.63	167.19	118.75	48.43	10.63	0.31
3	9.06	45.63	92.19	93.75	48.43	10.63	0.31
4	2.81	14.38	29.69	31.25	17.18	4.38	0.31
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0

1/ This Table was developed from Table 26, using equation (23).

Table 28 1/

$^{1}K_{Uij}$ in Dollars

Rounded to the nearest dollar.

* = less than .5

i \ j	1	2	3	4	5	6	7
1	-9853	-4907	-2016	-693	-161	-21	-*
2	-9302	-4857	-2016	-693	-161	-21	-*
3	-8525	-4532	-1941	-668	-161	-21	-*
4	-6328	-3563	-1566	-543	-129	-15	-*
5	-3224	-1844	- 817	-280	- 65	- 8	-*
6	0	0	0	0	0	0	0

1/ This Table was developed from Tables 17, 20, and 27, using equation (12). $U_{o;j}$ is of course zero for all j.

5. Costs for Expedited Repair Sub-Optimization

In this section, we shall develop $K_{U_{ij}}$ for the expedited repair sub-optimization.

Table 29 1/

${}^4U''_{sij}$ in Dollars

i \ j	-3	-2	-1	0	1	2
1	5,000	5,000	3,000	1,000	500	0
2	5,000	5,000	3,000	1,000	500	0
3	5,000	5,000	4,000	2,000	750	250
4	5,000	5,000	4,000	2,000	750	250
5	5,000	5,000	3,000	1,000	500	0

1/ This Table was developed from assumption 2 and Table 11, using equation (13).

Table 30 1/

${}^4U'_{sij}$ in Dollars

Rounded to nearest cent.

i \ j	1	2	3	4	5	6
1	500.00	0	0	0	0	0
2	550.00	50.00	0	0	0	0
3	1,007.50	357.50	52.50	2.50	0	0
4	1,288.18	494.83	116.95	15.05	0.98	0.03
5	994.20	325.35	67.40	8.50	0.60	0

1/ This Table was developed from Tables 7 and 29, and assumption 16, using equation (14).

Table 31 [1/]

$^{4}U_{sij}$ in Dollars

Rounded to the nearest cent.

i \ j	1	2	3	4	5	6
1	4,339.88	1,227.68	236.85	26.05	1.58	0.03
2	3,839.88	1,227.68	236.85	26.05	1.58	0.03
3	3,289.88	1,177.68	236.85	26.05	1.58	0.03
4	2,282.38	820.18	184.35	23.55	1.58	0.03
5	994.20	325.35	67.40	8.50	0.60	0

1/ This Table was developed from Table 30, using equation (15).

Table 32 [1/]

$^{4}H_{ij}$ (y) where i=2

j \ y	1	2	3	4	5
1	.50	0	0	0	0
2	.50	.50	0	0	0
3	0	.50	.50	0	0
4	0	0	.50	.50	0
5	0	0	0	.50	.50

1/ Tables 32 - 35 were developed from Tables 7 and 11; and assumptions 11 and 16; using equation (20).

Table 33

${}^4H_{ij}$ (y) where i=3

j \ y	1	2	3	4	5
1	.45	0	0	0	
2	.50	.45	0	0	
3	.05	.50	.45	0	
4	0	.05	.50	.45	
5	0	0	.05	.50	.45

Table 34

${}^4H_{ij}$ (y) where i=4

j \ y	1	2	3	4	5	6
1	.2025	0	0	0	0	0
2	.4500	.2025	0	0	0	0
3	.2950	.4500	.2025	0	0	0
4	.0500	.2950	.4500	.2025	0	0
5	.0025	.0500	.2950	.4500	.2025	0
6	0	.0025	.0500	.2950	.4500	.2025

Table 35

$^4H_{ij}$ where i=5

j \ y	1	2	3	4	5	6	7
1	.164025	0	0	0	0	0	0
2	.400950	.164025	0	0	0	0	0
3	.321975	.400950	.164025	0	0	0	0
4	.098100	.321975	.400950	.164025	0	0	0
5	.013975	.098100	.321975	.400950	.164025	0	0
6	.000950	.013975	.098100	.321975	.400950	.164025	0
7	.000025	.000950	.013975	.098100	.321975	.400950	.164025

Table 36 1/

$^4U''_{Bij}$ in Dollars

Rounded to the nearest cent.

i \ j	1	2	3	4	5	6	7
2	50.00	100.00	100.00	100.00	100.00	100.00	100.00
3	45.00	140.00	195.00	200.00	200.00	200.00	200.00
4	20.25	85.50	160.00	194.50	199.75	200.00	200.00
5	16.40	56.50	88.70	98.51	99.90	100.00	100.00
6	0	0	0	0	0	0	0

1/ This table was developed from assumption 16 and Tables 13, 32, 33, 34, and 35; using equation (21).

Table 37 1/

$^{4}U'_{Bij}$ in Dollars

Rounded to the nearest cent.
* = less than .005

i \ j	1	2	3	4	5	6	7
2	50.00	50.00	0	0	0	0	0
3	45.00	95.00	45.00	5.00	0	0	0
4	20.25	65.25	74.50	34.50	5.25	0.25	0
5	16.40	40.10	32.20	9.81	1.39	0.10	*
6	0	0	0	0	0	0	0

1/ This Table was developed from Table 36, using equation (22).

Table 38 1/

$^{4}U_{Bij}$ in Dollars

Rounded to the nearest cent.
* = less than .005

i \ j	1	2	3	4	5	6	7
1	131.65	250.35	151.70	49.31	6.64	0.35	*
2	81.65	200.35	151.70	49.31	6.64	0.35	*
3	36.65	105.35	106.70	44.31	6.64	0.35	*
4	16.40	40.10	32.20	9.81	1.39	0.10	*
5	0	0	0	0	0	0	0

1/ This Table was developed from Table 37, using equation (23).

Table 39 1/

$^4K_{Uij}$ in Dollars

Rounded to the nearest dollar.
* = less than .50

i \ j	1	2	3	4	5	6	7
1	-4488	-1481	-389	-75	-8	-*	-*
2	-3937	-1431	-389	-75	-8	-*	-*
3	-3340	-1286	-344	-70	-8	-*	-*
4	-2309	- 863	-217	-33	-3	-*	-*
5	- 999	- 327	- 68	- 9	-1	0	0
6	0	0	0	0	0	0	0

1/ This Table was developed from Tables 17, 31, and 38; using equation (12). $U_{o\ j}$ is of course zero for all j.

Table 40 1/

R'_k and $R'_{k,k-1}$ in Dollars

k	$\bar{x}$	R^e_k	R^r_k	R'_k and $R'_{k,k-1}$
1	.5	40	10	12
2	.5	40	10	12
3	1	40	0	32
4	1	40	0	32
5	.5	0	0	0
6	0	0	0	0

1/ This Table was developed from assumptions 6, 13, 14, 15, 16, 17, and 18, using equation (27).

6. The Routine Repair and Expedited Repair Sub-Optimizations

Figure 4 1/

Routine Repair Sub-Optimization

i	P_i	U_{i1}	U_{i2}	U_{i3}	U_{i4}	U_{i5}	U_{i6}	Total	Shadow Price
	1	1							
1	150	-9853	-4907	-2016	-693	-161	-21	0	150
	3		1	1	1				
2	90	-9302	-4857	-2016	-693	-161	-21	0	90
	1					1			
3	80	-8525	-4532	-1941	-668	-161	-21	0	80
	0								
4	70	-6328	-3563	-1566	-543	-129	-15	0	70
	0								
5	60	-3224	-1844	- 817	-280	- 65	- 8	0	60
	2						1		
6	0	0	0	0	0	0	0		
Total	7	1	1	1	1	1	1		
Shadow Price	0	-9703	-4767	-1926	-603	- 81	0		

Cost: $552.00

1/ In each cell, the entry below the line is the cost of the activity in dollars, drawn from Tables 14 and 28; the entry above the line is the activity level in the optimal (or sub-optimal) solution. The cost of the routine repair sub-optimization was obtained from Tables 17, 19, 26, and 40; and this figure; using equation (36).

Figure 5 1/

Expedited Repair Sub-Optimization

i	P_i	U_{i1}	U_{i2}	U_{i3}	U_{i4}	U_{i5}	Total	Shadow Price
	1	1						
1	150	-4488	-1481	-389	-75	-8	0	150
	2		1	1				
2	90	-3937	-1431	-389	-75	-8	0	90
	0							
3	80	-3340	-1286	-344	-70	-8	0	80
	0							
4	70	-2309	- 863	-217	-33	-3	0	70
	0							
5	60	- 999	- 327	- 68	- 9	-1	0	60
	4				1	1		
6	0	0	0	0	0	0		
Total	7	1	1	1	1	1		
Shadow Price	0	-4338	-1341	-299	0	0		

Cost: $532.00

1/ In each cell, the entry below the line is the cost of the activity, in dollars, drawn from Tables 14 and 39; the entry above the line is the activity level in the optimal (or sub-optimal) solution. The cost of the expedited repair sub-optimization was obtained from Tables 17, 30, 37, and 40, and this figure, using equation (36).

7. The Reverse Sub-Optimization

Since the expedited repair was the lower cost of the two, we shall perform the reverse sub-optimization on the procurement plan given by the expedited repair sub-optimization.

Table 41 1/

${}^{2}U''_{sij}$ in Dollars

i \ j	-3	-2	-1	0	1	2	3	4
1	5,000	5,000	3,000	1,000	500	0	0	0
2	5,000	5,000	4,000	2,000	750	250	0	0
3	5,000	5,000	4,500	3,000	1,375	500	125	0
4	5,000	5,000	4,750	3,750	2,187.50	937.50	312.50	62.50
5	5,000	5,000	4,500	3,000	1,375	500	125	0

1/ This Table was developed from assumption 2 and Table 9, using equation (13).

Table 42 1/

${}^{2}U'_{sij}$ in Dollars — Rounded to the nearest cent.

i \ j	1	2	3	4	5	6	7
1	500	0	0	0	0	0	0
2	750	250	0	0	0	0	0
3	1,537.50	587.50	162.50	12.50	0	0	0
4	2,494.38	1,190.63	443.75	116.25	14.38	0.63	0
5	2,014.14	891.50	305.89	66.00	8.01	0.50	0.01

1/ This Table was developed from Tables 5 and 41, and assumption 16, using equation (14).

Table 43 1/

${}^{3}U''_{sij}$ in Dollars

i \ j	-3	-2	-1	0	1	2	3
1	5,000	5,000	3,000	1,000	500	0	0
2	5,000	5,000	3,000	1,000	500	0	0
3	5,000	5,000	4,500	3,000	1,375	500	125
4	5,000	5,000	4,500	3,000	1,375	500	125
5	5,000	5,000	4,500	3,000	1,375	500	125

1/ This Table was developed from assumption 2 and Table 10, using equation (13).

Table 44 1/

${}^{3}U'_{sij}$ in Dollars Rounded to the nearest cent.

i \ j	1	2	3	4	5	6	7
1	500.00	0	0	0	0	0	0
2	550.00	50.00	0	0	0	0	0
3	1,537.50	587.50	162.50	12.50	0	0	0
4	1,857.88	784.13	252.75	45.25	3.88	0.13	0
5	2,014.14	891.50	305.89	66.00	8.01	0.50	0.01

1/ This Table was developed from Tables 6 and 43, and assumption 16, using equation (14).

Table 45 1/

$^2H_{ij}$ (y) where i=2

j \ y	1	2	3	4	5	6	7
1	.50	0	0	0	0	0	0
2	.50	.50	0	0	0	0	0
3	0	.50	.50	0	0	0	0
4	0	0	.50	.50	0	0	0
5	0	0	0	.50	.50	0	0
6	0	0	0	0	.50	.50	0
7	0	0	0	0	0	.50	.50

1/ Tables 45 - 48 were developed from Tables 5 and 9 and assumptions 11 and 16, using equation (20).

Table 46

$^2H_{ij}$ (y) where i=3

j \ y	1	2	3	4	5	6	7
1	.25	0	0	0	0	0	0
2	.50	.25	0	0	0	0	0
3	.25	.50	.25	0	0	0	0
4	0	.25	.50	.25	0	0	0
5	0	0	.25	.50	.25	0	0
6	0	0	0	.25	.50	.25	0
7	0	0	0	0	.25	.50	.25

Table 47

$^2H_{ij}$ (y) where i=4

j \ y	1	2	3	4	5	6	7
1	.1125	0	0	0	0	0	0
2	.3500	.1125	0	0	0	0	0
3	.3750	.3500	.1125	0	0	0	0
4	.1500	.3750	.3500	.1125	0	0	0
5	.0125	.1500	.3750	.3500	.1125	0	0
6	0	.0125	.1500	.3750	.3500	.1125	0
7	0	0	.0125	.1500	.3750	.3500	.1125

Table 48

$^2H_{ij}$ (y) where i=5

j \ y	1	2	3	4	5	6	7
1	.050625	0	0	0	0	0	0
2	.213750	.050625	0	0	0	0	0
3	.349375	.213750	.050625	0	0	0	0
4	.272500	.349375	.213750	.050625	0	0	0
5	.099375	.272500	.349375	.213750	.050625	0	0
6	.013750	.099375	.272500	.349375	.213750	.050625	0
7	.000625	.013750	.099375	.272500	.349375	.213750	.050625

Table 49 1/

${}^{2}U''_{Bij}$ in Dollars

Rounded to the nearest cent.

i \ j	1	2	3	4	5	6	7
2	50.00	100.00	100.00	100.00	100.00	100.00	100.00
3	25.00	100.00	175.00	200.00	200.00	200.00	200.00
4	11.25	57.50	130.00	182.50	198.75	200.00	200.00
5	5.06	26.44	61.38	88.63	98.56	99.94	100.00
6	0	0	0	0	0	0	0

1/ This Table was developed from assumption 16 and Tables 13, 45, 46, 47, and 48, using equation (21).

Table 50 1/

${}^{3}H_{ij}$ (y) where i=2

j \ y	1	2	3	4	5	6
1	.50	0	0	0	0	0
2	.50	.50	0	0	0	0
3	0	.50	.50	0	0	0
4	0	0	.50	.50	0	0
5	0	0	0	.50	.50	0
6	0	0	0	0	.50	.50

1/ Tables 50-53 were developed from Tables 6 and 10, and assumptions 11 and 16, using equation (20).

Table 51

${}^3H_{ij}$ (y) where i=3

j \ y	1	2	3	4	5	6
1	.45	0	0	0	0	0
2	.50	.45	0	0	0	0
3	.05	.50	.45	0	0	0
4	0	.05	.50	.45	0	0
5	0	0	.05	.50	.45	0
6	0	0	0	.05	.50	.45

Table 52

${}^3H_{ij}$ (y) where i=4

j \ y	1	2	3	4	5	6
1	.1125	0	0	0	0	0
2	.3500	.1125	0	0	0	0
3	.3750	.3500	.1125	0	0	0
4	.1500	.3750	.3500	.1125	0	0
5	.0125	.1500	.3750	.3500	.1125	0
6	0	.0125	.1500	.3750	.3500	.1125

Table 53

$^{3}H_{ij}$ where i=5

j \ y	1	2	3	4	5	6	7
1	.091125	0	0	0	0	0	0
2	.303750	.091125	0	0	0	0	0
3	.367875	.303750	.091125	0	0	0	0
4	.192500	.367875	.303750	.091125	0	0	0
5	.040875	.192500	.367875	.303750	.091125	0	0
6	.003750	.040875	.192500	.367875	.303750	.091125	0
7	.000125	.003750	.040875	.192500	.367875	.303750	.091125

Table 54 [1]

$^{3}U''_{Bij}$ in Dollars   Rounded to the nearest cent.

i \ j	1	2	3	4	5	6	7
2	50.00	100.00	100.00	100.00	100.00	100.00	100.00
3	45.00	140.00	195.00	200.00	200.00	200.00	200.00
4	11.25	57.50	130.00	182.50	198.75	200.00	200.00
5	9.11	39.49	76.28	95.53	99.61	99.99	100.00
6	0	0	0	0	0	0	0

[1] This Table was developed from assumption 16, and Tables 13, 50, 51, 52, and 53, using equation (21).

Table 55 1/

V_k and $V_{k,k-1}$ in Dollars   Rounded to the nearest cent.

k	1	2	3	4
$_{k+1}V^m_{sk}$	0	50.00	232.00	278.29
$_{k+2}V^m_{sk}$	50.00	150.00	278.29	0
$_{k+1}V^m_{s,k,k-1}$	—	10.00	115.20	65.42
$_{k+P_d+1}V^m_{Bk}$	20.00	30.00	29.40	0
$_{k+P_d+2}V^m_{Bk}$	30.00	14.50	0	0
$_{k+P_d+1}V^m_{Bk,k-1}$	—	30.00	27.32	0
V_k	-100.00	-244.50	-539.69	-278.29
$V_{k,k-1}$	—	-204.50	-420.81	- 65.42

1/ This Table was developed from Tables 19, 25, 30, 36, 42, 44, 49, and 54; and figure 5; using equations (28 through 35).

Table 56 1/

K_{R_k} and $K_{R_{k,k-1}}$ in Dollars   Rounded to the nearest dollar.

i	K_{R_k}	$K_{R_{k,k-1}}$
1	- 88	—
2	-233	-193
3	-508	-389
4	-246	- 33

1/ This Table was developed from Tables 40 and 55, using equations (25) and (26).

We may now proceed to the reverse sub-optimization, which is shown in Figure 6. The reverse sub-optimization has caused no change in policy which would differentiate it from the expedited repair sub-optimization; its cost is thus also $532.00.

Figure 6 [1]

Reverse Sub-Optimization

Activity Level	1		0	1			0	1			0	1				
Activity / Eq. No.	R_1	R_{10}	R_2	R_{21}	R_{20}	S_2	R_3	R_{32}	R_{30}	S_3	R_4	R_{43}	R_{40}	S_4	Total	Shadow Price
1	1	1													1	- 48
2			1	1	1										1	- 74
3							1	1	1						1	-176
4											1	1	1		1	- 33
5	1		1			1									1	- 40
6			1	1			1			1					1	-119
7							1	1			1			1	1	-213
Cost	-88	0	-233	-193	0	0	-508	-389	0	0	-246	-33	0	0		

1/ The costs of activities were obtained from Table 56.

V. SUMMARY OF TERMS USED

In this Section, we have listed and defined all of the terms used elsewhere in the paper. The terms are listed in alphabetical order, Roman characters before Greek characters, as follows:

Major sort:	prime character
1st intermediate:	post subscript
2nd intermediate:	post superscript
3rd intermediate:	pre-subscript
Minor:	pre-superscript

1. A = maximum amount of procurement,
2. A_j = maximum number of items which can be ordered in period j.
3. a_i = the cumulative expected gains of the spare item from attrition and whole aircraft salvage prior to repair period i.
4. a_B = the ratio of base stock-outs which lead to AOCP's to total base stock-outs.
5. a_s = the ratio of system stock-outs which lead to AOCP's to total system stock-outs.
6. a_i' = the cumulative expected gains of the spare item from attrition and whole aircraft salvage up to and including model period i.
7. $\bar{a}_k$ = the expected gain of spare items in model period k from attrited and salvaged aircraft.
8. b_B = the ratio of base stock-outs requiring maintenance cannibalization to all base stock-outs.
9. b_s = the ratio of system stock-outs requiring maintenance cannibalization to total system stock-outs.

10. $C_i(t)$ = the condemnation probability function for model periods prior to repair period i, $= \sum_{x=t}^{\infty} w_x(t)\ \bar{f}_i(x)$.

11. $C_i'(t)$ = the condemnation probability function for all periods in the model up to and including model period i, $= \sum_{x=t}^{\infty} w_x(t)\ f_i'(x)$.

12. c = the ratio of the number of days in the period of the time used in the model to the number of days in the maintenance flowtime of the aircraft.

13. d_j = the value of decision deferment in period j.

14. d' = a coefficient which is varied between runs in using Model 3, with a Monte Carlo model, to determine the value of decision deferment.

15. E_j = the experience in programmed flying hours of the applicable aircraft, up to and including ordering period j.

16. e_B = the cost of a base AOCP = the cost of the lost utility of the aircraft per day multiplied by the number of days in the priority depot-to-base pipeline.

17. e_s = the cost of a system AOCP lasting for one period of time.

18. $f_i(x)$ = the system demand probability function for model periods in repair period i.

19. $f_i^k(x)$ = the demand probability function in period i at base k = the probability of demanding x in period i at base k.

20. $f_i'(x)$ = the system demand probability function for all model periods up to and including model period i.

21. $\bar{f}_i(x)$ = the system demand probability function for model periods prior to repair period i.

22. $g_i(y)$ = the savings in base stockage in period i from having the y-th item in a serviceable condition in period $i - P_d$.

23. $H_{ij}(y)$ = the probability function of having y items in a serviceable condition in period $i - P_d$ if j items are delivered from procurement through period $i - P_d$.

24. h_k = the cost of holding one item in the system in period k.

25. ${}_a h_i$ = the cost in the i-th time period caused by holding an item that is in the system as a serviceable or reparable;

$$= h_i \, (1 - \Omega_1)\,(1 - \Omega_2)\,(1 - \Omega_3) \text{ ------ } (1 - \Omega_i)$$

26. K_{P_i} = the cost of activity P_i.

27. $K_{P_{ij}}$ = the cost of activity P_{ij}.

28. K_{R_k} = the cost of activity R_k.

29. $K_{R_{k,k-1}}$ = the cost of activity $R_{k,k-1}$.

30. $K_{U_{ij}}$ = the cost of activity U_{ij}.

31. M = the cost of one exchange of the spare item; that is, the cost of installing the spare item once and removing it once.

32. P_i = the amount procured for delivery in period i; (Model 1).

33. P_{ij} = the amount of procurement ordered in period j for delivery in period i; (Model 3).

34. ${}_b P_j$ = the true cost of procurement of the j-th item,

$$= P_{p,n-1} \sum_{t=0}^{j+a'_n-1} C'_n(t).$$

35. P_d = routine depot-base pipeline time.

36. P_{ei} = the expedited procurement penalty per item delivered in the i-th period.

37. P_{eij} = the expedited procurement penalty per item ordered in the j-th period for delivery in the i-th period.

38. P_{hi} = the lifetime cost of holding an item delivered in period i assuming that item does not suffer condemnation or obsolescence.

39. P_{pi} = the unit cost under routine procurement of an item ordered for delivery in period i.

40. R_k is equal to either 0 or 1; if it is equal to 1, repair is expedited on all reparables generated in the k-th period, when repair has not been similarly expedited in the preceding period; (Model 2).

41. R_k^e = the total cost per item of performing expedited repair on all reparables generated in period k.

42. R_k^r = the cost per item of performing routine repair on all reparables generated in period k. The value of R_k^r will be zero if k is within the routine repair cycle pipeline of the last active period of the model.

43. $R_{k,0}$ is equal to either 0 or 1; if it is equal to 1, repair is not expedited on reparables generated in the k-th period; (Model 2).

44. $R_{k,k-1}$ is equal to either 0 or 1; if it is equal to 1, repair is expedited on all reparables generated in the k-th period when repair has been similarly expedited in the preceding period; (Model 2).

45. R'_k = the additional costs incurred by expediting repair in period k, including therein, additional costs of packing, transporting, inspecting, handling, and overhauling.

46. $R'_{k,k-1} = R'_k$

47. S_k = a surplus activity in Model 2.

48. s_B = the cost of a base stock-out.

49. s_s = the cost of a system stock-out.

50. t = the number of condemnations; a random variable; see $C_i(t)$ and $C'_i(t)$.

51. U_{ij} equals either 1 or 0. If $U_{ij} = 1$, the j-th item is procured for delivery in the i-th period.

52. U_{Bij} = the savings in base stockage costs from buying the j-th item in the i-th period.

53. U_{hij} = the holding cost refund associated with buying the j-th item in the i-th period.

54. U_{oj} = the obsolescence refund associated with buying the j-th item, = the refund of part of the procurement cost of the j-th item to represent the probability that that item will be totally consumed (condemned) and thus not incur terminal obsolescence.

55. U_{sij} = the savings in system depletions from buying the j-th item in the i-th period.

56. U'_{Bij} = the savings in period i in base stockage costs from buying the j-th item for delivery in or before the $i - P_d$th period.

57. U'_{sij} = savings to be obtained in period i from buying the j-th item for delivery in or before period i.

58. ${}^{1}U'_{sij}$ = savings to be obtained in period i from buying the j-th item for delivery in or before the i-th period if routine repair is used for all reparables generated in periods $i - 2$ and $i - 1$.

59. ${}^{2}U'_{sij}$ = savings to be obtained in period i from buying the j-th item for delivery in or before the i-th period if expedited repair is used in period $i - 2$ and routine repair is used in period $i - 1$.

60. ${}^{3}U'_{sij}$ = savings to be obtained in period i from buying the j-th item for delivery in or before the i-th period if routine repair is used in period $i - 2$, and expedited repair is used in period $i - 1$.

61. ${}^{4}U'_{sij}$ = savings to be obtained in period i from buying the j-th item for delivery in or before the i-th period if repair is expedited in both period $i - 2$ and $i - 1$.

62. U''_{Bij} = the total savings in period i in base stockage costs from buying j items (items 1 through j) for delivery in or before the $i - P_d$th period.

63. ${}^{1}U''_{Bij}$ = the total savings in period i in base stockage costs from buying j items for delivery in or before the $i - P_d$th period, if routine repair is used in periods $i - P_d - 2$ and $i - P_d - 1$.

64. ${}^{2}U''_{Bij}$ = the total savings in period i in base stockage costs from buying j items for delivery in or before the $i - P_d$th period, if expedited repair is used in period $i - P_d - 2$ and routine repair is used in period $i - P_d - 1$.

65. ${}_{3}U''_{Bij}$ = the total savings in period i in base stockage costs from buying j items for delivery in or before the $i - P_d$th period, if routine repair is used in period $i - P_d - 2$, and expedited repair is used in period $i - P_d - 1$.

66. ${}_{4}U''_{Bij}$ = the total savings in period i in base stockage costs from buying j items for delivery in or before the $i - P_d$th period, if repair is expedited in both periods $i - P_d - 2$ and $i - P_d - 1$.

67. U''_{sij} = savings to be obtained in period i from having the j-th item in the system that is not condemned prior to repair period i.

68. V_k = the savings in reduced expected cost of system depletions and reduced base stockage costs from expediting repair in period k, if repair is not also expedited in period $k-1$.

69. $V_{k,k-1}$ = the savings in reduced expected cost of system depletions and reduced base stockage costs from expediting repair in period k, if repair is also expedited in period $k-1$.

70. ${}_{k+P_d+1}V^{m}_{Bk}$ = the savings in base stockage costs in period $k+P_d+1$ from expediting repair in period k, if repair is not expedited in period $k-1$, and if m items are delivered from procurement through period $k+1$.

71. ${}_{k+P_d+2}V^{m}_{Bk}$ = the savings in base stockage costs to be obtained in period $k+P_d+2$ by expediting repair on reparables generated in period k, if m items are delivered from procurement through period $k+2$.

72. ${}_{k+P_d+1}V^m_{Bk,k-1}$ = the savings in base stockage costs to be obtained in period $k+P_d+1$ by expediting repair in period k if repair has been expedited on reparables generated in period $k-1$, and if m items are delivered from procurement through period $k+1$;

73. ${}_{k+1}V^m_{sk}$ = the saving in system depletions to be obtained in period $k+1$ by expediting repair on reparables generated in period k when repair is not expedited on reparables generated in period $k-1$, and if m items are delivered from procurement through period $k+1$.

74. ${}_{k+2}V^m_{sk}$ = the savings in system depletions to be obtained in period $k+2$, if m items are delivered from procurement through period $k+2$.

75. ${}_{k+1}V^m_{sk,k-1}$ = the savings in system depletions to be obtained in period $k+1$ by expediting repair on reparables generated in period k, if repair is also expedited on reparables generated in period $k-1$, and if m items are delivered from procurement through period $k+1$.

76. $w_x(t)$ = the condemnation probability function given x issues, =

$$(1-\bar{w})^{x-t}\,\bar{w}^{t}\,\frac{x!}{t!(x-t)!}$$

77. $\bar{w}$ = the probability of an item received in exchange being condemned, = wearout factor or per cent.

78. x = the number of items demanded; a random variable; see $f_i(x)$, $f_i^k(x)$, $f_i'(x)$ and $\bar{f}_i(x)$

80. y = the number of serviceable items in the system; a random variable; see $g_i(y)$.

81. Π_B = the base depletion penalty

82. Π_{Bq} = the cost of the q-th base depletion to occur in one model period, at one base.

83. Π_s = the system depletion penalty.

84. Π_{sq} = the cost of the q-th system depletion to occur in one model period.

85. σ_i = the cumulative deliveries from procurement through period i.

86. Ω_k = the probability of engineering change obsolescence in period k.